Department of Mathematics

F.Y.B.Sc.(Computer Science)

Question Bank

Paper–I:*Graph Theory*

Answer in One Sentence(or in 2 – 3 lines) **(2 marks questions)**

1. Define the term:Continuous Function.
2. State Rolle's Mean Value Theorem.
3. Is every continuous function always differentiable ? justify.
4. If f(x)=$\frac{1}{x}$ $x \neq 0$ then show that f(x) is not continuous at x≠0.
5. Verify Rolle's mean value theorem for f(x)=(x-1)(x-2)(x-3) on [0,3].
6. State Lagrange's Mean Value Theorem.
7. Discuss the applicability of LMVT to the function$f(x) = x^{\frac{1}{3}}$,x ∈[-1,1].
8. Show that f(x)=$x^3 - 3x^2 + 3x + 2$ is strictly (monotonically) increasing in every interval.
9. State Cauchy's Mean Value Theorem.
10. Write any four indeterminate forms.
11. Find second derivative of Y=$\frac{1}{x}$.
12. State Leibnitz's Theorem.
13. Find derivative of Y=cos (bx +c).
14. State Taylor's theorem with Lagrange's form of remainder.
15. State Taylor's theorem with Lagrange's form of remainder.
16. State Taylor's theorem with Cauchy's form of remainder.
17. State intermediate value theorem.
18. State fixed point property.
19. Give an example of unbounded continuous function on non-closed interval with justification.
20. Find the n^{th} derivative of e^x.
21. Evaluate $\lim_{x\to 0} \frac{2^x - 1}{3^x - 1}$
22. Define Total Derivative.
23. Define Differential Equations.
24. Define Ordinary Differential Equation.
25. Determine the order and degree of differential equation

$$x\left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right)^4 + y^4 = 0.$$

Short Answer Questions **(4 marks questions)**

1. Discuss the continuity of f(x),
   where f(x)=$\frac{x^2-9}{x+3}$; x≠-3
   $=\frac{3}{2}$; x=-3.
2. Discuss continuity of the function f(x) defined as:

   f(x)= $\begin{cases} -x, & 0 \le x < \frac{1}{2} \\ 1-x, & \frac{1}{2} \le x < 1 \end{cases}$.

3. Find value of ɗ if function is continuous in (-2,1)
   f(x)=4x+5 if -2< $x < 0$
   =2x+ɗ if 0≤ $x < 1$.
4. Show that Rolle's theorem is not applicable for the function,f(x) =|x|,x∈[-1,1].
5. Verify the Rolle's theorem for the function f(x)=$x^2(1-x)^2$ in [0,1].
6. State and prove Lagrange's mean value theorem.
7. Verify the Lagrange's mean value theorem for the function f(x) = log x in [1,e].
8. Using Lagrange's mean value theorem show that,$\frac{1}{8} \le \sqrt{51}$ -$\sqrt{49} < \frac{1}{7}$.
9. Verify the Cauchy's mean value theorem for f(x)=sin x and g(x)=cos x in [0,$\frac{\Pi}{2}$].
10. Evaluate: $\lim_{x\to 0} \frac{a^x-b^x}{x}$
11. Evaluate:$\log_{x\to 0}(x^2 \log x)$
12. State and prove Rolle's mean value theorem.
13. Find the n^{th} derivative of Y=sin $bx + c$.
14. If Y=$e^x \sin x + 3$, then find Y_n.
15. Find n^{th} derivative of : y $= \frac{1}{x^2+3x+2}$.
16. State and prove Leibnitz's theorem for the n^{th} derivative of product of two function.
17. Find n^{th} derivative of y=$e^x \log x$, by using Leibnitz's theorem.
18. Find n^{th} derivative of y=$x^3 \cos x$.
19. If y =$\sin^{-1} x$, then prove that , (1-x^2)y_{n+2}-(2n+1) xy_{n+1} -$n^2 y_n$=0.
20. If x=tan($logy$) , the show that, (1+x^2)y_{n+1}+(2nx-1)y_n+n(n-1) =0.
21. If y =$\frac{1}{x^2-x-2}$ $find\, y_n$.
22. Expand $\sec x$ by Maclaurin's theorem as far as the term x^4.
23. Assuming the validity of expansion , obtain the series expansion of $\tan x$ in ascending power of x up to x^5 term.
24. Expand, log (1+x) in ascending power of x.
25. Show that, $e^{\sin x}$=$1 + x + \frac{x^2}{2} - \frac{x^4}{8} + \cdots$.

26. Write Maclaurin's series for f(x) = e^{3x} and find $f^8(0) = ?$
27. Expand e^{e^x} in ascending power of x.
28. Assuming the validity of expansion, expand $e^x \sin x$ in ascending power of (x -$\frac{\Pi}{4}$) up to five terms.
29. Prove that log(x + h) = log h + $\frac{x}{h}$ - $\frac{x^2}{2h^2}$+.... .
30. If f(x) is a polynomial of degree 3 such that, f(1) =1, f'(1)=3, f'' (1) =5, f''' $(1) = 3$ then find f(3).
31. Expand , $3x^3 - 2x^2 + x - 4$ in ascending powers of (x-3).
32. Evaluate:$\lim_{x\to 0} \frac{a^x - b^x}{x}$
33. Find n^{th} derivative of y=sin $3x \cos 2x$.
34. If y = asin $px + b\cos px$; then prove that ; $y_2 + p^2 y = 0$.
35. Determine the order and degree of the differential equation
$y = px + \sqrt{a^2p^2 + b^2}$, where,p= $\frac{dy}{dx}$
36. Solve $\frac{dy}{,dx} = \sin(x + y)$
37. Solve , $(x^2 + xy)\frac{dy}{dx} = 2xy$.
38. Solve, $(x^2 - 2xy - y^2)$dx-$(x + y)^2 dy$=0.
39. Solve,(1+xy)y dx+(1-xy)x dy=0.
40. Solve (1+x^2)$\frac{dy}{dx} + 2xy = \cos x$.
41. Solve,$x\sqrt{1 - y^2}dx + y\sqrt{1 - x^2}\,dy = 0$.
42. Solve, $(x^4 + y^4)$ dx –$xy^3\,dy = 0$.
43. Solve, $\cos x \cos y\, dy + \sin x \sin y\, dy = 0$.
44. Solve, $(x^2y - 2xy^2)$ dx - $(x^3 - 3x^2y)dy = 0$.
45. Solve, $(1 + 6y^2 - 3x^2y)\,\frac{dy}{dx} = 3xy^2 - x^2$.

Long Answer Questions **(8 marks questions)**

1. Find value of α and β$\begin{cases} 4x + 5 & if -2 < x < 0 \\ 2x + \alpha & if \quad 0 \le x < 1 \\ x - 3\beta & if \quad 1 \le x < 3 \end{cases}$
2. State and prove Rolle's Mean Value Theorem. Explain geometric interpretation of Rolle's Theorem.
3. State and prove Lagrange's Mean Value Theorem. Explain geometric interpretation of Lagrange's Theorem.
4. State and prove Cauchy's Mean Value Theorem.
5. If $a < 1, b < 1\ and\ b > a$ then prove that:

$\frac{b-a}{\sqrt{1-a^2}} < \sin^{-1} b - \sin^{-1} a < \frac{b-a}{\sqrt{1-b^2}}$.

6. Prove that,
   $\frac{b-a}{1+b^2} < \tan^{-1} b - \tan^{-1} a < \frac{b-a}{1+a^2}$, if $a < b$.

7. State Leibnitz Theorem, if $y = \sin^{-1} x$ then prove that,
   $$(1-x^2)y_{n+2} - 2(n+1)xy_{n+1} - n^2 y_n = 0.$$
8. Using Lagrange's mean value theoremshow that $\frac{1}{8} \leq \sqrt{51} - \sqrt{49} < \frac{1}{7}$.
9. Evaluate: $\left(\lim_{x\to 0} \frac{x\cos x - \log(1+x)}{x^2}\right)$
10. Evaluate: $\left[\left(\lim_{x\to\frac{\pi}{2}} \cos x \cdot \log(\tan x)\right)\right]$
11. State and prove Leibnitz's theorem and hence find y_n for y=$x^3 \cdot \sin x$.
12. If y=$\sin^{-1} x$ then prove that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - n^2 y_n = 0$
13. State Leibnitz's theorem and hence prove that if
    y=$\sin(m \sin^{-1} x)$ then $(1-x^2)y_{n+2}$=(2n+1)xy_{n+1}+$(n^2-m^2)y_n$.
14. Find n^{th} derivative of
    $$y = \frac{1}{(x-1)(x-2)(x-3)}$$
15. Expand, $\log(\sin x)$ in ascending power of (x-3).
16. Prove that: $\sin^{-1} x = x + 1^2 \cdot \frac{x^3}{3!} + 1^2 \cdot 3^2 \cdot \frac{x^5}{5!} + \cdots$.
17. Expand $\log\left[\tan\left(x + \frac{\Pi}{4}\right)\right]$ in ascending power of x.
18. Expand $(1+x)^x$ in ascending power of x.
19. If $y^{1/m} + y^{-1/m} = 2x$,prove that ,
    $(x^2-1)\, y_{n+2}$+(2n+1)xy_{n+1}+$(n^2-m^2)y_n$=0
20. Find value of α and β $\begin{cases} 3x-7 & if -2 < x < 0 \\ 2x+\alpha & if \quad 0 \leq x < 1 \\ x - 3\beta & if \quad 1 \leq x < 3 \end{cases}$
21. Examine the continuity of f defined on ℝ where
    f(x) = 1 if x is rational
    = -1 if x is irrational
22. Discuss the continuity of f(x),
    where f(x)=2x-1 if 0< $x \leq 1$
    = x^2 if 1< $x < 2$
    =3x-4 if 2≤ $x \leq 4$
    =$x^{3/2}$ if x≥ 4
23. Solve,(x-y)dx+(x+y)dy=0.
24. Solve, (2x+y+3)dx=(2y+x+1)dy.
25. Solve,$(y^2 + 2x^2 y)$ dx+ $(2x^3 - xy)$ dy =0.

26. Solve, $y - x\frac{dy}{dx} = a\left(y^2 + \frac{dy}{dx}\right)$.

27. Solve, $(x^3y^3 + xy)\frac{dy}{dx} = 1$

28. Solve, $y + 2\frac{dy}{dx} = y^3(x - 1)$.

29. Solve, $(x^3 + xy^4)$ dx $-2y^3\,dy = 0$.

30. Solve, $\frac{dy}{dx} = \frac{2x+9y-20}{6x+2y-10}$.