# Anekant Education Society's <br> Tuljaram Chaturchand College of Arts, Science and Commerce, Baramati <br> Autonomous <br> Course Structure for S.Y.B.Sc. Mathematics (2022 Pattern) 

S. Y. B. Sc. Mathematics

| Semester | Course <br> Code | Title of Course | No. of <br> Credits | No. of <br> Lectures |
| :---: | :--- | :--- | :---: | :---: |
| III | USMT231 | Calculus of Several Variables | 3 | 48 |
|  | USMT232 | Laplace Transform \& Fourier Series | 3 | 48 |
|  | USMT233 | Practical based on USMT231 and <br> USMT232 | 2 | 48 |
|  | USMT241 | Vector Calculus | 3 | 48 |
|  | USMT242 | Linear Algebra | 3 | 48 |
|  | USMT243 | Practical based on USMT241 and <br> USMT242 | 2 | 48 |

Equivalence of the old syllabus with the new syllabus
Sem IV

| Old Course |  | New Course |  |
| :--- | :--- | :--- | :--- |
| MAT2401 | Multivariable Calculus-II | USMT241 | Vector Calculus |
| MAT2402 | Linear Algebra | USMT242 | Linear Algebra |
| MAT2403 | Practical Based on <br> MAT2401 \& MAT2402 | USMT243 | Practical based on <br> USMT241 and USMT242 |

## Academic Year 2023-24

Class: S.Y.B.Sc. (Semester - IV)
Course Code: USMT 241
Course: 1
Title of the Course: Vector Calculus
Credit: 2
No. of Lectures: 48

## A) Learning Objectives:

- To develop a solid understanding of double integrals over rectangles, including the techniques for evaluating them and their geometric interpretations.
- To explore the concept of iterated integrals, enabling students to decompose complex regions and integrate over them using both the horizontal and vertical orders of integration.
- To extend the knowledge to double integrals over general regions, allowing students to apply various coordinate transformations and choose appropriate integration bounds for non-rectangular domains.
- Develop a deep understanding of triple integrals, enabling students to calculate volumes, masses, and other quantities in three-dimensional space.
- Attain proficiency in utilizing cylindrical and spherical coordinates to simplify and solve complex integration problems, expanding the scope of applications in threedimensional calculus.
- Understand the fundamental concepts and properties of vector fields, including vector operations, divergence, curl, and the interpretation of vector fields in physical contexts.
- Develop proficiency in computing line integrals, applying various techniques to evaluate path-dependent quantities in vector fields, and interpreting their significance in real-world applications.
- Develop a solid comprehension of the concepts of curl and divergence in vector fields.
- Learn methods for computing surface areas of parametric surfaces and apply them in diverse contexts.
- Understand the principles of Stoke's theorem and the divergence theorem, and be able to apply them to evaluate line integrals, flux, and volume integrals.


## B) Learning Outcomes:

- Students will be able to compute double integrals over rectangles, demonstrating proficiency in setting up and evaluating integrals in Cartesian coordinates for various functions and geometric regions.
- Students will have the capability to apply polar coordinates for double integrals, enabling them to analyze and evaluate functions in circular and sectorial regions, and understand the advantages of this coordinate system in certain scenarios.
- Students will be able to set up and evaluate triple integrals over general regions in Cartesian, cylindrical, and spherical coordinates, demonstrating proficiency in solving problems related to volume, mass, and other physical quantities.
- Students will demonstrate the ability to seamlessly transition between Cartesian, cylindrical, and spherical coordinate systems, selecting the most appropriate system
for a given problem. They will apply these skills to solve a range of real-world problems, including those involving irregularly shaped objects and regions.
- Students will be able to analyze and manipulate vector fields, demonstrating a solid grasp of vector operations, divergence, and curl, and their respective applications in physics and engineering.
- Upon completion of the course, students will be capable of effectively utilizing line integrals to calculate quantities such as work, circulation, and flux, and will understand the geometric interpretations and practical implications of these computations.
- Students will be able to identify and calculate curl and divergence for various vector fields, enabling them to analyze the behavior of physical systems governed by these vector fields.
- Students will demonstrate proficiency in computing areas of parametric surfaces and will be able to apply Stoke's theorem and the divergence theorem to solve problems related to flux, circulation, and volume integrals.


## TOPICS/CONTENTS:

## Unit 1: Double Integrals

(12 Lectures)
1.1 Double integrals over rectangles
1.2 Iterated integrals
1.3 Double integrals over general regions
1.4 Double integrals in polar coordinates
1.5 Applications of double integrals

## Unit 2: Triple Integrals

(12 Lectures)
2.1 Triple integrals
2.2 Triple integrals in cylindrical coordinates
2.3 Triple integrals in spherical coordinates
2.4 Change of variables in multiple integrals
2.5 Applications of triple integrals

## Unit 3: Line Integrals

(12 Lectures)
3.1 Vector fields
3.2 Line integrals
3.3 The fundamental theorem for line integrals
3.4 Green's theorem

## Unit 4: Surface Integrals

(12 Lectures)
4.1 Curl and divergence
4.2 Parametric surfaces and their areas
4.3 Surface integrals
4.4 Stoke's theorem
4.5 The divergence theorem

## Text Book:

James Stewart, Calculus with Early Transcendental Functions, Cengage Learning, Indian Edition
Unit 1 - Sections 15.1 to 15.5 , Unit 2 - Sections 15.6 to 15.9 ,
Unit 3 - Sections 16.1 to 16.4, Unit 4 - Sections 16.5 to 16.9.

## Reference Books:

1. G. B. Thomas, Thomas' Calculus, Pearson, Edition 2012.
2. Tom M. Apostol, Calculus Vol. II, John Wiley.
3. Shanti Narayan and R. K. Mittal, A text-book of Vector Calculus, S. Chand and Company.
4. J. E. Marsden, A. J. Tromba and A. Weinstein, Basic Multivariable Calculus, Springer.
5. D. V. Widder, Advanced Calculus, Printice Hall of India.

## Academic Year 2023-24

Class: S.Y.B.Sc. (Semester - IV)
Course Code: USMT 242
Course: 1 Title of the Course: Linear Algebra
Credit: 2

## A) Learning Objectives:

- To make the students become familiar with the basic concepts of linear algebra with a thorough understanding of vector spaces, linear transformations and inner product spaces.
- To understand the concept of matrices, linear system and solutions of systems.
- To Construct and interpret linear transformations in $\mathbb{R}^{2}$ or $\mathbb{R}^{3}$.
- To make the students aware of the crucial importance of linear algebra to many fields in engineering, statistics and computer science.
- To compute inner products on a real vector space and compute angle and orthogonality in inner product spaces.
- To learn Rank and nullity of linear transformations through matrices.
- To understand the axiomatic structure of a modern mathematical subject and learn to construct simple proofs.


## B) Learning Outcomes:

## By the end of the course, students will be able to:

- Determine if a system of linear equations has no solution, one solution, or infinitely many solutions.
- Find the null space of a matrix and span of independent vectors.
- Find the matrix of a linear transformation given bases of relevant vector spaces.
- Use Linear Algebra as a powerful tool for computations.
- Learn properties of inner product spaces and determine orthogonality in inner product spaces.
- Visualize the space in $\mathbb{R}^{n}$ terms of vectors and their interrelation with matrices.
- Apply this knowledge to many fields in engineering, statistics and computer science.


## TOPICS/CONTENTS:

## Unit-1: Matrices and System of Linear Equations

1.1 Row echelon form of a matrix, reduced row echelon form of a matrix.
1.2 Rank of a matrix
1.3 System of linear equations
1.4 Consistency of homogeneous and non-homogeneous system of linear equations.
1.5 Solution of System of Equations: Gauss elimination and Gauss-Jordan elimination method.

## Unit-2: Vector Spaces

(16 Lectures)
2.1 Real Vector Spaces
2.2 Subspaces.
2.3 Linear Dependence and Independence.
2.4 Basis of Vector Space
2.5 Dimension of a Vector Space.
2.6 Row, Column and Null Space of a matrix.
2.7 Rank and nullity.

Unit-3: Linear Transformations
(14 Lectures)
3.1 Definition and Examples, Properties, Equality.
3.2 Kernel and range of a linear Transformation
3.3 Rank-Nullity theorem.
3.4 Composite and Inverse Transformation.
3.5 Matrix of Linear Transformation.
3.6 Linear Isomorphism.

Unit-4: Inner Product Spaces (12 Lectures)
4.1 Definitions and Examples
4.2 Angle and Orthogonality in Inner Product Spaces
4.3 Gram-Schmidt Process

## Text Book:

## Howard Anton, Chris Rorres, Elementary Linear Algebra, Application Version, Ninth Edition, Wiley, 11th edition.

Unit-1: Chapter-1: Sec. 1.1, 1.2.
Unit-2: Chapter-4: Sec. 4: 4.1 to 4.5, 4.7 and 4.8
Unit-3: Chapter-8:Sec. 8.1to 8.4
Unit-4: Chapter-6: Sec 6.1 to 6.3

## Reference Books:

1. K. Hoffman and R. Kunze, Linear Algebra, 2nd edition (2014), Prentice Hall of India, New Delhi
2. Steven J. Leon, Linear Algebra with Applications, 4th edition (1994), Prentice Hall of India.New Delhi
3. Vivek Sahai, Vikas Bist, Linear Algebra, 4th Reprint 2017, Narosa Publishing House, New Delhi
4. Promode Kumar Saikia, Linear Algebra, 2009, Pearson, Delhi
5. S. Lang, Introduction to Linear Algebra, 2nd edition, 1986, Springer-Verlag, New York, Inc.

Class: S.Y. B. Sc. (Sem- IV)Title of Paper: Practical based on USMT241 and USMT242
Paper Code: USMT243 Credit: 2
Paper: III
No. of lectures: 48

## Learning Objectives:

1. Understanding the Concepts of Double and Triple Integrals: Learn the fundamental principles of double and triple integrals, including their geometric interpretations and how they relate to volume and area calculations.
2. Application of Coordinate Transformations: Explore different coordinate systems, such as polar and spherical coordinates, and understand how to apply them in the context of multiple integrals.
3. Analyzing Vector Fields and Line Integrals: Gain proficiency in analyzing vector fields, understanding their significance in physics and engineering, and being able to calculate line integrals over vector fields.
4. Applying Divergence and Stoke's Theorems: Learn the divergence theorem and Stoke's theorem and apply them to solve real-world problems involving flux, circulation, and surface integrals.
5. Matrix Operations and Linear Equations: Understand matrix operations, including row echelon form and Gauss elimination, and their role in solving systems of linear equations.
6. Exploring Vector Spaces and Independence:Learn the fundamental concepts of vector spaces, subspaces, linear dependence, and independence, as well as how to identify and work with bases.
7. Understanding Linear Transformations: Grasp the definition of linear transformations, their properties, and their representation through matrices, and understand the ranknullity theorem.
8. Analyzing Inner Product Spaces and Orthogonality: Learn about inner product spaces, including definitions, examples, and how to determine angles and orthogonality in these spaces.

## Learning Outcomes:

1. Competence in Integration Techniques: Students will be able to perform double and triple integrals, calculate area and volume in various coordinate systems, and relate these integrals to real-world applications.
2. Mastery of Coordinate Transformations: Students will be proficient in transforming between coordinate systems, making calculations in polar and spherical coordinates, and applying these skills in a range of problems.
3. Proficiency in Vector Field Analysis: Students will be capable of understanding and analyzing vector fields, making connections to physical phenomena, and calculating line integrals over these fields.
4. Problem-Solving Using Advanced Theorems: Students will be able to apply the divergence theorem and Stoke's theorem to solve complex engineering and physics problems, showcasing their ability to apply vector calculus principles in practical contexts.
5. Solving Linear Equations: Students will be able to apply matrix operations to solve systems of linear equations, classify the types of solutions, and perform Gauss elimination and Gauss-Jordan elimination method.
6. Proficiency in Vector Space Concepts: Students will be capable of analyzing and working with vector spaces, identifying subspaces, linear dependence, independence, bases, and computing the rank and nullity.
7. Application of Linear Transformations: Students will be proficient in understanding linear transformations, calculating the kernel and range, applying the rank-nullity theorem, and representing linear transformations through matrices.
8. Analysis of Inner Product Spaces: Students will be able to apply the concepts of inner product spaces to determine angles and orthogonality in practical contexts, making connections to applications in physics and engineering.

## Title of experiments:

## Vector Calculus:

1. Exploring Double Integrals: From Rectangles to Polar Coordinates
2. Triple Integrals and Coordinate Transformations
3. Vector Fields and Line Integrals
4. Surface Integrals and Divergence Theorems
5. Advanced Topics in Vector Calculus
6. Applications of Vector Calculus in Engineering and Science

## Linear Algebra:

1. Matrix Operations and Solving Linear Equations
2. Vector Spaces and Linear Dependence
3. Linear Transformations and Isomorphisms
4. Inner Product Spaces and Orthogonality
5. Applications of Linear Algebra in Engineering and Computer Science
6. Dimension and Basis in Vector Spaces

# Mapping of Program Outcomes with Course Outcomes 

Class: SYBSc (Sem IV)
Course: Vector Calculus
Weightage: $1=$ weak or low relation, $2=$ moderate or partial relation, $3=$ strong or direct relation

|  | Programme Outcomes (POs) |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Course <br> Outcomes | PO 1 | PO 2 | PO 3 | PO 4 | PO 5 | PO 6 | PO 7 | PO 8 | PO 9 |
| CO 1 | 3 | 3 |  |  |  |  |  |  |  |
| CO 2 | 3 | 3 |  |  | 1 |  |  | 1 |  |
| CO 3 | 3 | 3 |  | 1 | 1 |  |  | 1 |  |
| CO 4 | 3 | 2 |  |  |  |  |  |  |  |
| CO 5 | 3 | 3 |  |  |  |  |  |  |  |
| CO 6 | 3 | 3 |  |  |  |  |  |  |  |
| CO 7 | 3 | 3 |  |  |  |  |  |  |  |
| CO 8 | 3 | 3 |  |  |  |  |  |  |  |

## Justification for the mapping

## PO1: Disciplinary Knowledge

CO1: Student will demonstrate proficiency in setting up and evaluating double integrals over rectangles in Cartesian coordinates, showcasing their ability to apply disciplinary knowledge to compute areas and volumes for various functions and geometric regions.
CO2: Mastering polar coordinates for double integrals equips student to proficiently analyze circular and sectorial regions, providing a powerful tool for mathematical modeling and problem-solving in diverse scientific and engineering fields.
CO3: Student will gain the ability to apply advanced mathematical techniques in Cartesian, cylindrical, and spherical coordinates to solve real-world problems involving volume, mass, and other physical quantities, showcasing their mastery of Disciplinary Knowledge.
CO4: Student will master diverse coordinate systems, facilitating informed choices between Cartesian, cylindrical, and spherical systems for efficient problem-solving, particularly in realworld scenarios with complex geometries, enhancing their disciplinary knowledge.
CO5: Student will acquire a proficient understanding of vector operations, divergence, and curl, enabling them to effectively analyze and manipulate vector fields for practical applications in physics and engineering disciplines.
CO6: Student will master line integrals, grasping both their geometric significance and practical utility across disciplines, by engaging in rigorous mathematical coursework that equips them to calculate work, circulation, and flux with precision.
CO7: Student can identify and calculate curl and divergence for vector fields, facilitating the analysis of physical systems by providing insights into their rotational and divergent properties.
CO8: Mastering parametric surfaces, Stoke's theorem, and the divergence theorem equips student with the tools to effectively analyze flux, circulation, and volume integrals, enhancing their grasp of advanced vector calculus concepts within disciplinary knowledge.

## PO2: Critical Thinking and Problem Solving

CO1: Student will develop the ability to apply critical thinking and problem-solving skills in computing double integrals over rectangles, demonstrating proficiency in setting up and
evaluating integrals in Cartesian coordinates for diverse functions and geometric regions, enhancing their mathematical problem-solving capabilities.
CO 2 : Student will acquire the ability to apply polar coordinates for double integrals, facilitating the analysis and evaluation of functions in circular and sectorial regions, thereby enhancing their problem-solving skills by harnessing the advantages of this coordinate system in scenarios that exhibit radial symmetry.
CO3: Student will develop a versatile mathematical skillset, enabling them to efficiently analyze complex three-dimensional geometries and apply integral calculus to solve real-world problems involving volume, mass, and other physical quantities.
CO4: Student will master coordinate system transitions to tackle real-world problems effectively by choosing the most suitable system, enhancing their critical thinking and problem-solving abilities.
CO5: Studying vector fields enhances critical thinking and problem-solving skills by enabling student to proficiently analyze and manipulate vector operations, divergence, and curl, essential for tackling complex problems in physics and engineering.
CO6: Studying line integrals enhances students' analytical thinking by providing them with a powerful mathematical tool to quantify and understand physical phenomena, such as work, circulation, and flux, allowing for deeper insight into real-world applications and geometric interpretations.
CO7: Studying curl and divergence equips student with essential tools to analyze and understand the dynamic behavior of vector fields, enhancing their critical thinking and problem-solving abilities in the realm of physical systems.
CO8: Student will demonstrate proficiency in computing areas of parametric surfaces and applying Stoke's theorem and the divergence theorem to solve problems related to flux, circulation, and volume integrals in order to develop advanced problem-solving skills essential for real-world applications in various fields.

## PO4: Research-related skills and Scientific temper

CO3: Mastering triple integrals in diverse coordinates enhances precision in complex geometry analysis, enabling accurate calculations of volume, mass, and crucial physical properties for research, nurturing a robust scientific mindset.

## PO5: Trans-disciplinary Knowledge

CO2: Mastering polar coordinates for double integrals empowers student to proficiently analyze functions in circular and sectorial regions, providing a versatile tool applicable across various disciplines for more efficient problem-solving and analysis.
CO3: Student will acquire the ability to apply triple integrals in various coordinate systems, enabling them to analyze complex regions and calculate volume, mass, and diverse physical properties across disciplines.

## PO8: Environment and Sustainability

CO2: Proficiency in applying polar coordinates for double integrals empowers student to effectively analyze and evaluate functions in circular and sectorial regions, providing a crucial tool for addressing environmental and sustainability challenges that often exhibit inherent rotational symmetry.
CO3: Proficiency in setting up and evaluating triple integrals in various coordinate systems enables student to accurately analyze and quantify volume, mass, and other essential physical parameters, facilitating informed decision-making for environmental and sustainability concerns.

Choice Based Credit System Syllabus (2022 Pattern)
Mapping of Program Outcomes with Course Outcomes

Class: S.Y.B.Sc. (Sem IV)
Course: Linear Algebra

Subject: Mathematics
Course Code: USMT242

Weightage: $1=$ weak or low relation, $2=$ moderate or partial relation, $3=$ strong or direct relation.

|  | Programme Outcomes(POs) |  |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Course <br> Outcomes | PO 1 | PO 2 | PO 3 | PO 4 | PO 5 | PO 6 | PO 7 | PO 8 | PO 9 |
| CO 1 | 3 | 3 |  | 2 |  | 2 |  |  | 2 |
| CO 2 | 3 | 3 |  |  |  | 2 |  |  | 2 |
| CO 3 | 3 | 3 |  |  |  | 2 |  |  | 2 |
| CO 4 | 3 | 3 |  | 2 | 3 | 2 |  |  | 2 |
| CO 5 | 3 | 3 |  | 2 |  | 2 |  |  | 2 |
| CO 6 | 3 | 3 |  |  |  | 2 |  |  | 2 |
| CO 7 | 3 | 3 |  | 2 | 1 | 2 |  |  | 2 |

## Justification for the mapping

PO 1: Disciplinary Knowledge:
All of these course outcomes (COs) contribute to the development of students disciplinary knowledge in mathematics.For example, $\mathrm{CO} 1, \mathrm{CO} 2, \mathrm{CO} 3, \mathrm{CO} 5, \mathrm{CO} 6$ requires student to develop deep learning of vector spaces,linear transformation, inner product spaces. CO4 requires students to apply the concepts of linear algebra in many fields like engineering,statistics and computer science.

## PO2:Critical Thinking and Problem Solving:

All of these course outcomes (COs) contribute to the development of students critical thinking and problem solving. For example, $\mathrm{CO} 1, \mathrm{CO} 2 \mathrm{CO} 3, \mathrm{CO} 5$ requires students to think critically and apply these to solve complex problems in various filed like engineering and physics. CO4,CO6 and CO7 requires to apply and construct logical proofs to solve real world problems.
PO4: Research-related skills and Scientific temper:
$\mathrm{CO} 1, \mathrm{CO} 4, \mathrm{CO}, \mathrm{CO} 7$ contribute to the development of students research related skills and scientific temper. For example, CO 4 and CO 7 requires students to develop their ability to think critically and apply knowledge to various field.CO1 and CO5 requires students to apply knowledge of system of linear equations and inner product spaces and apply to solve real world problem.

## PO5:Trans-disciplinary Knowledge:

CO4, CO7 requires students to apply linear algebra tools in various fields like Physics, Engineering and Computer science.

## PO6:Personal and professional competence:

All COs contribute to development of personal and professional competences. For example , all COs requires students to approach and solve complex problem systematically.
PO9:Self-directed and Life-long learning:
All these course outcomes contribute to development of students ability to engage in self directed and life-long learning. For example, all COs requires students to develop their ability to learn new concepts, form a simple proof and apply them to new problem.

## Choice Based Credit System Syllabus (2022 Pattern)

## Mapping of Program Outcomes with Course Outcomes

Class: SYBSc (Sem II)
Course: Practical based on USMT241 and USMT242
Weightage: $1=$ weak or low relation, $2=$ moderate or partial relation, $3=$ strong or direct relation

|  | Programme Outcomes (POs) |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Course <br> Outcomes | PO 1 | PO 2 | PO 3 | PO 4 | PO 5 | PO 6 | PO 7 | PO 8 | PO 9 |
| CO 1 | 3 | 3 |  |  |  |  |  |  |  |
| CO 2 | 3 | 3 |  | 2 | 2 | 2 |  |  | 2 |
| CO 3 | 3 | 3 |  | 2 | 2 | 2 |  |  | 2 |
| CO 4 | 3 | 3 |  |  | 2 | 2 |  |  | 2 |
| CO 5 | 3 | 3 |  |  |  | 2 |  |  | 2 |
| CO 6 | 3 | 3 |  |  |  | 2 |  |  | 2 |
| CO 7 | 3 | 3 |  |  | 2 | 2 |  |  | 2 |
| CO 8 | 3 | 3 |  | 2 | 2 | 2 |  |  | 2 |

## Justification for the mapping

## PO1: Disciplinary Knowledge:

All of the course outcomes (COs) contribute to the development of students' disciplinary knowledge in mathematics. For example, $\mathrm{CO} 1, \mathrm{CO} 2$, and CO 3 require students to master advanced integration techniques, coordinate transformations, and vector field analysis. CO4 and CO5 require students to apply these concepts to complex problems in engineering and physics. CO6 and CO7 require students to develop a deep understanding of vector spaces and linear transformations. And CO8 requires students to apply the concepts of inner product spaces to practical problems.

## PO2: Critical Thinking and Problem Solving

All of the COs also contribute to the development of students' critical thinking and problem-solving skills. For example, $\mathrm{CO} 1, \mathrm{CO} 2$, and CO 3 require students to think critically about how to apply different integration techniques and coordinate transformations to solve problems. CO 4 and CO 5 require students to use their knowledge of vector field analysis and advanced theorems to solve complex engineering and physics problems. CO6 and CO7 require students to think critically about abstract concepts such as vector spaces and linear transformations. And CO8 requires students to apply their knowledge of inner product spaces to solve realworld problems.

## PO4: Research-related skills and Scientific temper

$\mathrm{CO} 2, \mathrm{CO} 3$, and CO 8 contribute to the development of students' research-related skills and scientific temper. For example, CO 2 requires students to learn how to use mathematical software to perform complex calculations. CO3 requires students to develop their ability to think critically about experimental data and formulate hypotheses. And CO8 requires students to apply their knowledge of inner product spaces to solve problems in physics and engineering.

## PO5: Trans-disciplinary knowledge

$\mathrm{CO} 2, \mathrm{CO} 3, \mathrm{CO} 4, \mathrm{CO}$, and CO 8 contribute to the development of students' trans-disciplinary knowledge. For example, CO 2 requires students to learn how to apply mathematical concepts to problems in engineering and physics. CO 3 requires students to develop an understanding of the connections between mathematics and other disciplines, such as physics and engineering. CO4 and CO7 require students to apply their knowledge of vector calculus and linear algebra to solve complex problems in a variety of fields. And CO8 requires students to apply the concepts of inner product spaces to practical problems in physics and engineering.

## PO6: Personal and professional competence

$\mathrm{CO} 2, \mathrm{CO} 3, \mathrm{CO} 4, \mathrm{CO}, \mathrm{CO}, \mathrm{CO}$, and CO 8 all contribute to the development of students' personal and professional competence. For example, all of the COs require students to develop their ability to work independently and as part of a team. They also require students to develop their communication skills and their ability to apply their knowledge to solve real-world problems.

## PO9: Self-directed and Life-long learning

$\mathrm{CO} 2, \mathrm{CO} 3, \mathrm{CO} 4, \mathrm{CO}, \mathrm{CO}, \mathrm{CO}$, and CO 8 all contribute to the development of students' ability to engage in self-directed and life-long learning. For example, all of the COs require students to develop their ability to learn new concepts and apply them to new problems. They also require students to develop their ability to think critically about their own learning and to identify areas where they need to improve.

