Anekant Education Society's

Tuljaram Chaturchand College of Arts, Science and Commerce, Baramati

Autonomous

Course Structure for M.Sc. Mathematics (2022 Pattern)

M.Sc Mathematics-Semester IV

Semester	Course Code	Title of Course	No. of Credits	No. of Lectures
III	PSMT231	Combinatorics	4	64
	PSMT232	Field Theory	4	64
	PSMT233	Functional Analysis	4	64
	PSMT234	Integral Equations	4	64
	PSMT235(A)	Astronomy	4	64
	PSMT235(B)	Graph Theory	4	64
	PSMT236	Practical: Python	4	64
IV	PSMT241	Number Theory	4	64
	PSMT242	Differential Geometry	4	64
	PSMT243	Fourier Analysis	4	64
	PSMT244	Lattice Theory	4	64
	PSMT245(A)	Coding theory	4	64
	PSMT245(B)	Cryptography	4	64
	PSMT246	Project	4	64

Academic Year 2023-24 M.Sc.-II

Class	**: M.Sc.-II (Semester – IV)**		
Course Code	**: PSMT241**		
Course	**: I**	**Title of the Course**	**: Number Theory**
Credit	**: 4**	**No. of lectures**	**: 64**

A) Course Objectives

- To impart the knowledge of encryption and decryption techniques and their applications in managing the security of data.
- To express the concept and results of number theory effectively.
- To learn various arithmetic functions, Sigma function, Algebraic Numbers, and congruence relation.
- To identify and characterize prime numbers, and recognize their significance.
- To Learn about divisibility rules, and prime factorization.
- To give elementary ideas from number theory this will have applications in various fields.
- To introduce the concepts of Quadratic reciprocity.

B) Course Outcomes

- Student will be able to find the quotients and remainders from integer division.
- Student will be able to understand the definitions of congruence, residue classes and least residues.
- Student will be able to identify arithmetic functions and Dirichlet multiplications.
- Student will be able to establish existing identities using Mobius inversion formula.
- Student will be able to determine multiplicative inverses modulo n, and use to solve linear congruence.
- Student will be able to apply the Wilsons theorem and calculate primitive roots.
- Student will be able to understand the concepts of legendry symbol and identify the Quadratic or non-Quadratic residues modulo p.

TOPICS/CONTENT

Unit 1: Divisibility **[12 Lectures]**

1.1 Divisibility in integers
1.2 Division algorithm
1.3 G.C.D, L.C.M
1.4 Fundamental theorem of arithmetic
1.5 The number of primes
1.6 Mersene numbers and Fermat numbers

Unit 2: Congruences **[15 Lectures]**

2.1 Properties of congruence relation
2.2 Residue classes their properties Fermat's and Euler's theorems
2.3 Wilson's Theorem
2.4 Linear congruence of degree one
2.5 Chinese remainder theorem

Unit 3: Arithmetic functions **[10 Lectures]**

3.1 Euler function

3.2 Greatest integer function
3.3 Divisor function δ(n)
3.4 Mobius function μ(n)
3.5 Properties and their inter relation

Unit 4: Quadratic Reciprocity **[15 Lectures]**

4.1 Quadratic residue.
4.2 Legendre's symbol its properties
4.3 Quadratic Reciprocity law
4.4 Jacobi symbol its properties
4.5 Sums of Two Squares

Unit 5: Some Diophantine Equations **[04 Lectures]**

5.1 The equation ax + by = c
5.2 simultaneous linear equations

Unit 6: Algebraic numbers **[08 Lectures]**

6.1 Algebraic Numbers.
6.2 Algebraic number fields.
6.3 Algebraic integers.
6.4 Quadratic fields.
6.5 Units in Quadratic fields.
6.6 Primes in Quadratic fields.

Text Book:

Ivan Niven & H. S. Zuckerman, An introduction to number theory, (Wiley EasternLimited).

Unit 1: Sections 1.1 to 1.3
Unit 2: Sections 2.1 to Section 2.4
Unit 3: Section 3.1, 3.3, 3.6.
Unit 4: Section 4.1 to Section 4.4
Unit 5: Section 5.1 and Section 5.2
Unit 6: Section 9.1 to Section 9.7

Reference Book:

1. T.M. Apostol, An Introduction to Analytical Number Theory, Springer International Student's Edition.
2. David M Burton, Elementary Number Theory, Universal Book Stall, New Delhi.
3. S. G. Telang, Number Theory, Tata Mc-graw Hill.
4. W. Rudin, Functional Analysis, Tata McGraw Hill.
5. G. H. Hardy, E.M.Wright, Introduction to Number Theory, Oxford university press.

Academic Year 2023-24 M.Sc.-II

Class	**: M.Sc.-II (Semester – IV)**		
Course Code	**: PSMT242**		
Course	**: II**	**Title of the Course**	**: Differential Geometry**
Credit	**: 4**	**No. of lectures**	**: 64**

A) Course Objectives:

- To introduce equivalence of two curve, definition and parameterization of surface.
- To introduce tangent space of surfaces.
- To introduce integrate differential forms on surfaces.
- To get introduced to the notion of Serret-Frenet frame for space curves.
- To understand the idea of orientable and non-orientable surfaces.
- To get introduced to the concepts of a regular parameterized curve.
- To understand the isometry between two surfaces and characterization of local isometry between them.

B) Course Outcomes:

- Student will be able to understand the treatment of Level sets, Geodesics, weingarten map, smooth curve, and line integral.
- Student will be able to find differential maps between surfaces.
- Students will develop understanding of basics of differential geometry.
- Student will be able to understand and solve problems which require the use of differential geometry.
- Students will know how to use formal mathematical reasoning and write mathematical proofs when necessary.
- Students will demonstrate ability to cover a topic independently and tp present their results in a written report.
- Student will be able to perform calculations of curvature and related quantities for curves and surfaces in 3-dimensional spaces.

TOPICS/CONTENT

Unit 1: Graphs and Level Sets **[03 Lectures]**

1.1 Level set
2.1 Graphs of Level Sets

Unit 2: Vector Field **[12 Lectures]**

2.1 Dot product
2.2 Cross product.
2.3 Length of vector.
2.4 Vector Field, Smooth vector Field.
2.5 Gradient
2.6 Parameterized Curve.

Unit 3: The Tangent Space **[04 Lectures]**

3.1 Tangent to Level Sets
3.2 Properties

Unit 4: Surface and Vector field on surface **[14 Lectures]**

4.1 Surface of Revolution
4.2 Vector Field
4.3 Tangent Vector Field, Smooth Vector Field
4.4 Normal Vector Field
4.5 Connectedness

Unit 5: The Gauss Map and Geodesics **[8 Lectures]**

5.1 Gauss Map.
5.2 Spherical Image of Oriented n-Surfaces
5.3 Speed of α
5.4 Geodesics Property

Unit 6: The Parallel Transport, The Weingarten Map **[15 Lectures]**

6.1 Vector Field
6.2 Covariant Derivatives
6.3 Euclidean Parallel
6.4 Levi–civita
6.5 Use of parallelism
6.6 Properties of directional derivative
6.7 Covariant Derivative ofTangent vector field

Unit 7: Curvature of Plane Curve and Arc Length And Line Integral **[8 Lectures]**

7.1 Significance of sign of k(p)
7.2 Global Parameterization
7.3 Arc Length
7.4 Fundamental Domain,
7.5 Differentiable 1-form

<u>Text Book:</u>

J.A.Thorpe, Elementary Topics in Differential Geometry, Springer verleg.

<u>Reference Book:</u>

1. B Oneill, Elementary Differential Geometry, Acedamic New-York.
2. Do Carmo M., Differential Geometry of Curves and Surfaces, Englewood Cliffs, N. J. PrenticeHall, 1977.

Academic Year 2023-24 M.Sc.-II

Class	**: M.Sc.-II (Semester – IV)**		
Course Code	**: PSMT243**		
Course	**: III**	**Title of the Course**	**: Fourier Analysis**
Credit	**: 4**	**No. of lectures**	**: 64**

A) Course Objectives:

- To understand theory in Fourier analysis.
- To enable the students to study finite Fourier sine and cosine series.
- To introduce theory of differentiation and integration of Fourier series.
- Use Fourier series to solve boundary value problems.
- Understand the convergence of Fourier series of continuous periodic functions.
- To introduce Strum-Liouville problems.
- To understand the convergence of Fourier series of piecewise continuous functions.

B) Course Outcomes:

- Student will be able to calculate Fourier series of a function.
- Student will Classify and solve partial differential equations.
- Student will be able to evaluate the Fourier series expansion for different periodic functions.
- Student will discuss the nature of the partial differential equations.
- Student will be able to analyze the properties of a Fourier Transforms.
- Student will be able to calculate the Fourier sine and cosine series and apply it in solving boundary value problems.
- Student will be able to calculate the infinite Fourier series of elementary functions from the definition.

TOPICS/CONTENTS:

Unit 1: Fourier series **[10 lectures]**

1.1 Piecewise continuous functions
1.2 Fourier cosine series
1.3 Fourier sine series
1.4 Examples
1.5 Fourier series
1.6 Examples
1.7 Adaptations to other intervals

Unit 2: Convergence of Fourier Series **[12 lectures]**

2.1 One sided derivatives
2.2 A property of Fourier coefficients
2.3 A Fourier theorem
2.4 Discussion of the theorems and its corollary
2.5 Convergence on other intervals
2.6 Absolute and uniform convergence of Fourier series

2.7 Differentiation of Fourier series
2.8 Integration of Fourier series

Unit 3: The Fourier Method **[10 lectures]**
3.1 Linear operators
3.2 Principle of superposition
3.3 A temperature problem
3.4 A vibrating string problem

Unit 4: Boundary Value Problems **[10 lectures]**
4.1 A slab with faces at prescribed temperature
4.2 Related problems
4.3 A slab with internally generated heat
4.4 Steady temperatures in a rectangular plate
4.5 Cylindrical coordinates
4.6 A string with prescribed initial conditions

Unit 5: Orthonormal sets **[08 lectures]**
5.1 Inner products and orthonormal sets
5.2 Examples
5.3 Generalized Fourier series
5.4 Examples
5.5 Best approximation in the mean

Unit 6: Sturm-Liouville Problems and Applications **[08 lectures]**
6.1 Regular Strum Liouville problems
6.2 Orthogonality of eigen functions
6.3 Real valued eigen functions and nonnegative eigen values
6.4 Methods of solution
6.5 Examples of eigen-function Expansions
6.6 A temperature problem in rectangular coordinates

Unit 7: Bessel Functions and Applications **[06 lectures]**
7.1 Bessel functions $\boldsymbol{J_n(x)}$
7.2 General solutions of Bessel's equation
7.3 Recurrence relations
7.4 Bessel's integral form
7.5 Some consequences of the integral forms
7.6 The zeros of $\boldsymbol{J_n(x)}$

Text Book:

Churchill and Brown, Fourier Series and Boundary Value Problems, McGraw-Hill, 7th edition.

Reference Books:

E. Stein and R. Shakharchi, Fourier Series and Boundary Value Problems, New age International.

Academic Year 2023-24

Class : M.Sc.-II (Semester – IV)
Course Code : PSMT244
Course : IV **Title of the Course : Lattice Theory**
Credit : 4 **No. of lectures : 64**

A) Learning Objectives:

- To develop the concept of students in modern and universal algebra related with order and relation.
- To familiarize the concepts of poset, chain conditions.
- Generalization of lattice concept by dropping one or more of the lattice identities.
- To know the concept and applications of Lattice Theory.
- To study relation between Graph Theory and Lattice Theory
- To know Lattice-ordered Groups and related concepts.
- To study complements, relative complements and semi-complements of elements of a bounded lattices.

B) Learning Outcomes

- Student will be able to understand how lattices and Boolean algebra are used as tools and mathematical models in the study of networks.
- Students will able to classified Distributive and Modular Lattices.
- Student will be able to learn the equivalent conditions for a lattice to become modular and distributive.
- This will help the students to understand the concepts of bound elements, atoms and dual atoms in lattices.
- Student will be able to learn the property of homomorphism of lattices.
- To recognize the significance of ideal lattices.
- Students will be able to explain the relation between Graph Theory and Lattice Theory.

TOPICS/CONTENT

Unit 1: Lattice First Concepts **[36 Lectures]**

1.1 Two definitions of lattices
1.2 Hasse diagrams
1.3 Homomorphism
1.4 Isotone maps
1.5 Ideals and congruence relations
1.6 Congruence lattices
1.7 Product of lattices
1.8 Complete lattice
1.9 Ideal lattice
1.10 Distributive –Modular inequalities and identifies
1.11 Complements and pseudo complements
1.12 Boolean lattice of pseudo complements
1.13 Join and meet-irreducible elements.

Unit 2: Characterization theorems and representation theorems **[20 Lectures]**

2.1 Characterization theorem
2.2 Birkhoff`s distributivity criterion
2.3 Hereditary subsets, rings of sets
2.4 Stone theorems
2.5 Nachbin theorem
2.6 Statements of Hashimoto’s theorem.

Unit 3: Modular and Semimodular lattices **[8 Lectures]**

3.1 Isomorphism theorem
3.2 Upper and lower covering conditions
3.3 Kuros-Ore theorem
3.4 Jordan-Holder chain condition.

Text Book:

G. Gratzer, General Lattice Theory, Birkhauser, IInd Edition 1998.

Unit 1 – Sections 1.1, 1.2, 1.3, 1.4,1.6
Unit 2 – Section 2.1
Unit 3 – Section 3.1
Unit 4 – Section 3.2

Reference Books:

1. Lattice Theory: First Concepts and Distributive Lattices, George Gratzer.
2. Lattice Theory: Special Topics and applications, G. A. Gratzer, Fwehrung Springer.

Academic Year 2023-24 M.Sc.-II

Class	**: M.Sc. II (Semester – IV)**		
Course Code	**: PSMT 245(A)**		
Course	**: V(A)**	**Title of the Course**	**: Coding Theory**
Credit	**: 4**	**No. of lectures**	**: 64**

A) Learning Objectives:

- To learn how codes in mathematics are used for error correction and data transmission.
- To understand information theoretic behaviour of a communication system.
- To understand various source coding techniques for data compression.
- To understand various channels coding techniques and their capability.
- To build and understanding of fundamental concepts of data communication and networking.
- Development and implementation of advanced algorithms.
- To define and apply the basic concepts of information theory like entropy, channel capacity etc.

B) Learning Outcomes:

- Student will be able to derive equations for entropy, mutual information and channel capacity for all kinds of channels.
- Student will be able to implements the various types of source coding algorithms and analyse their performance.
- Student will be able to explain various methods of generating and detecting different types of error correcting codes.
- Student will be able to perform information theoretic analysis of communication system.
- Student will be able to design a data compression scheme using suitable source coding techniques.
- Student will be able to design a channel coding scheme for a communication system.
- Student will be able to comprehend various error control code properties.

TOPICS/CONTENT

Unit 1: Source Coding **[16 Lectures]**

1.1 Definition and examples
1.2 Uniquely decodable codes
1.3 Instantaneous codes
1.4 Constructing instantaneous codes
1.5 Kraft's inequality
1.6 McMillan's inequality

Unit 2: Optimal Codes **[16 Lectures]**

2.1 Optimality
2.2 Binary Huffman codes

2.3 Average word length of Huffman codes
2.4 Optimality of binary Huffman codes
2.5 R-ary Huffman codes
2.6 Extensions of sources

Unit 3: Entropy **[16 Lectures]**

3.1 Information and entropy
3.2 Properties of a entropy function
3.3 Entropy and average word length
3.4 Shannon- Fano Coding
3.5 Entropy of extensions and products
3.6 Shannon's first theorem
3.7 An example of Shannon's first theorem

Unit 4: Information channels **[16 Lectures]**

4.1 Notation and definitions
4.2 The binary symmetric channel
4.3 System entropies
4.4 Extension of Shannon's first theorem to information channels
4.5 Mutual information
4.6 Channel capacity

<u>Text Book:</u>

Gareth A. Jones and J. Mary Jones, Information and Coding Theory, Springer

Unit 1 – Sections 1.1 to 1.6
Unit 2 – Sections 2.1 to 2.6
Unit 3 – Sections 3.1 to 3.7
Unit 4 – Sections 4.1 to 4.8

<u>Reference Books:</u>

1. Andre Neubauer, Jurgen Freudenberger, Volker Kuhn, Coding Theory, Wiley.
2. S. Veluswamy, Information Theory and Coding, New Age International (P) Ltd.
3. J. H. van Lint, Introduction to Coding Theory, Springer, 3rd Edition.
4. P. S. Satyanarayana, Concept of Information Theory & Coding, Medtech.

Academic Year 2023-24 M.Sc.-II

Class	**: M.Sc.-II (Semester – IV)**		
Course Code	**: PSMT245 (B)**		
Course	**: V(B)**	**Title of the Course**	**: Cryptography**
Credit	**: 4**	**No. of lectures**	**: 64**

A) Course Objectives:

- To understand basics of Cryptography and various electronic codes.
- To learn different encryption techniques along with digital signatures and their use in various protocols.
- To learn about how to maintain the Confidentiality, Integrity and availability of a data.
- To develop attitude and interest along with necessary knowledge and skills among the students
- Explain the importance and application of each of confidentiality, integrity, Authentication and availability.
- To learn different encryption techniques using RSA algorithms.
- To impart the knowledge of encryption and decryption techniques and their applications in managing the security of data.

B) Course Outcomes:

- Student will be able to, apply some early substitution and translation ciphers.
- Student will be able to, distinguish symmetric key encryption systems from public key encryption systems.
- Student will be able to, assess simple cryptographic methods from a practical viewpoint.
- Student will be able to use cryptography methods to do further academic studies and research.
- Student will be able to perform Security Related real world problems.
- Student will demonstrate the use of symmetric key encryption systems and public key encryption systems.
- Students will be able to describe advantages and disadvantages of various encryption and decryption systems.

TOPICS/CONTENT

Unit 1: Introduction to cryptography **[17 Lectures]**

1.1 Cryptography in Modern world

1.2 Substitution cipher

1.3 Monoalphabetic ciphers

1.4 Transposition Cipher

1.5 Vigenere Cipher
1.6 Introduction to polygraphic substitution ciphers
1.7 cryptanalysis of substitution cipher

Unit 2: Symmetric key cryptography **[17 Lectures]**

2.1 Introduction and overview
2.2 Stream Cipher, Block cipher
2.3 Modes of operation Electronic code book
2.4 Cipher block chaining, Cipher feedback
2.5 Algorithms: Data Encryption Standard, Advanced Encryption Standard, IDEA (International Data Encryption Algorithm)
2.6 Attacks against DES, AES, IDEA

Unit 3: Public key Cryptography **[15 Lectures]**

3.1 Introduction and Overview
3.2 The RSA algorithm
3.3 Diffie Hellman Key protocol, exchange message
3.4 Algorithms: Discrete Logarithm, MD5
3.5 Attacks against RSA

Unit 4: Applications of Cryptography **[15 Lectures]**

4.1 Digital Signature
4.2 Kerberos
4.3 Pretty Good privacy
4.4 Internet protocol security
4.5 C, C++, and Python programming implementation of topics on Ciphers

<u>Reference Books:</u>

1. Adam J. Elbirt, Understanding and Applying cryptography and Data security, CRC press.
2. Bruice Schneier, Applied Cryptography, Wiley India Edition.
3. Atul Kahate, Cryptography and Network security, Tata Mcgraw Hill.
4. Neil Koblitz, A course in Number theory and Cryptography, Springer, Second Edition.