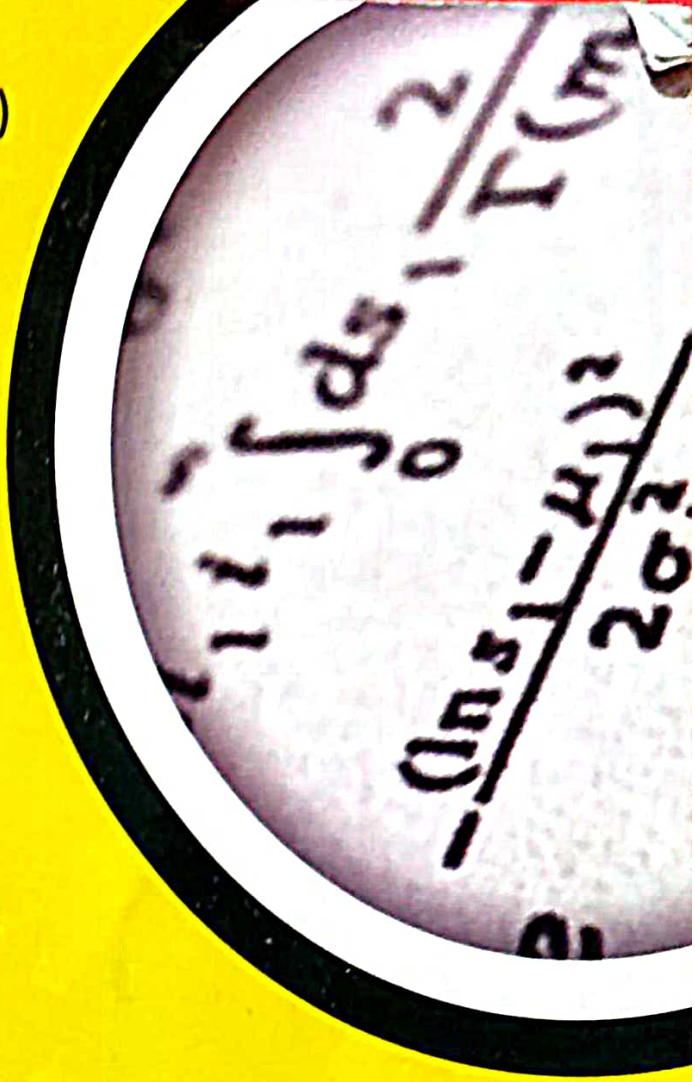
F.Y. B.Sc. (Computer Science)

A Text Book of

Mathematics-1 (MTC 121)

Linear Algebra

Prof. Arun S. Pandhari



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Semester II

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F.Y.B.Sc. Computer Science

Sem - II Paper - I

[MTC- 121]

Linear Algebra

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Dedicated To

My Mother

Nalini Tai (B.1931)

Who always supports me in any situation.

PREFACE

It gives me a great pleasure and satisfaction in presenting this text book on 'Linear Algebra' for F.Y.B.Sc. (Computer Science) Mathematics (2019 pattern) as per CBCS, SPPU, Pune for Sem II.

This book covers an entire syllabus and is written in keeping the needs of the students. It is useful to the professors teaching the subject. It is written according to the guidelines of the Board of Studies in Mathematics of the University. The matter is presented in a simple and understandable matter. The language is simple. A lot of solved examples of all types are covered. Exercises are given at the end of each subtopic.

The book is divided into 4 chapters - Vector Spaces, Eigenvalues and EigenVectors, Orthogonality and Symmetric Matrices and Affine Geometry of Vector Spaces. All these are useful for Computer Science students. The matter is written according to the prescribed text book so the notations are also of the same type.

Vision Publications always gives me freedom and support to write a good number of books for various standards including reference books also. These are widely accepted by the students and Professors community. The staff of Vision has taken great efforts for publishing this text book.

I also thank my wife Prof. Mrs. Anjali and My son Dr. Abhijit (Canada) who are a constant source of energy for me. I am also thankful to the management of Avekant Education Society and our Principal Dr. C. v. Murumkar for continuous support. Finally I am thankful to my Department colleagues - especially Prof. S. R. Puranik (Head).

Finally, I welcome the suggestions and criticism from all the readers.

Author

Prof. A. S. Pandhari

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to this book

Theorem 2

If $u_1, u_2, \ldots, u_r$ are eigenvectors corresponding to distinct eigenvalues $\lambda_1, \lambda_2, \ldots, \lambda_r$ of n n matrix A, then the set $\{u_1, u_2, \ldots, u_r\}$ is linearly independent.

Proof

Suppose $\{u_1, u_2, \ldots, u_r\}$ is linearly dependent. Then one of the vectors in the set is a linear combination of the preceding vectors. Let q be the least index such that u_{q+1} is a linear combination of the preceding vectors which are linearly independent.

Important theorems are given with proof.

Determine if the set $W = \{(x, y) / x^2 - y^2 = 0\}$ is a subspace of a vector space R^2.

Solution

No.

Since $W = \{(x, y) / x^2 - y^2 = 0\} \subseteq R^2$

But, for $\bar{u} = (x_1, y_1), \bar{v} = (x_2, y_2) \in W$

$\Rightarrow x_1^2 - y_1^2 = 0, x_2^2 - y_2^2 = 0$

and $\bar{u} + \bar{v} = (x_1 + x_2, y_1 + y_2)$

$(x_1 + x_2)^2 - (y_1 + y_2)^2 = (x_1^2 - y_1^2) + (x_2^2 - y_2^2) + 2(x_1 x_2 - y_1 y_2)$

$= 2(x_1 x_2 - y_1 y_2) \neq 0$

$\therefore \bar{u} + \bar{v} \notin W$ $\therefore$ W is not a subspace of R^2.

More solved examples are given for practice.

Note i. $X \neq 0$ but λ may be 0.

ii. By a transformation $X \rightarrow AX$, we have AX to be same λX. So X is transformed to λX, which is a scalar multiple of X.

Important notes are given in related topics.

Remark

1. λ is an eigenvalue of A $\Leftrightarrow$ The equation $(A - \lambda I) X = 0$ has a non-trivial solution. The set of all solutions of $(A - \lambda I) X = 0$ is a null space of the matrix $A - \lambda I$. So this is a subspace of IR^n called the 'eigen space' of A corresponding to λ. But note that it contains a 0 vector and all eigenvectors corresponding to λ.

Important remark is given where needed.

Unit 1: Vector Spaces (10 lecture

1.1 Vector spaces and subspaces

1.2 Null spaces, column spaces and linear tranformations.

1.3 Linearly independent sets : Bases

1.4 Coordinate systems

1.5 The dimension of a vector space

1.6 Rank

Unit 2: Eigen values and Eigen vectors (10 lectur

2.1 Eigen values and Eigen vectors

2.2 The characteristic equation

2.3 Diagonalization

2.4 Eigen vectors and Linear transformations

Unit 3: Orthogonality and Symmetric Matrices (10 lectu

3.1 Inner product, length and orthogonality

3.2 Orthogonal sets

3.3 Orthogonal Projections

3.4 Diagonalization of Symmetric Matrices

3.5 Quadratic forms

Unit 4: The Geometry of vector spaces (6lectu

4.1 Affine combinations

4.2 Affine independence

4.3 Convex combinations

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CONTENTS

4. The Geometry of Vector Spaces

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About Author

Mr. A.S. Pandhari has worked as P.G. Head of Mathematics at Tuljaram Chaturchand College, Baramati (Pune). He has a long experience of 38 years of teaching Mathematics and Statistics from U.G. to P.G. classes. He is resident and past student of T.C. College. The college is Accredited A+ by NACC last year. He did his M.Sc. from University of Pune, B.Ed. from Regional College of Education Bhopal. He stood first at B.Ed. He has passed M.Ed., M.Phil from University of Pune. Has consistently first class career. His topics of interest are Pure Mathematics, Statistics and Education. He has written 12 textbooks at F.Y./S.Y. B.Sc and B.C.S. students on Calculus, Algebra, Operations Research, etc. Published more than 30 articles in Marathi on various popular Science subjects in Newspaper and in science magazines in Marathi. He has worked in the fields of Adult Education and Competitive Examination Guidance. He has a long experience of working on various University /State Govt. Committees.

F.Y.B.Sc. Computer Science
(Sem-II)

- **C-I :** Advanced 'C' Programming
- **C-II :** Relational Database Management Systems
- **E-I :** Instrumentation Systems
- **E-II :** Basics of Computer Organisation
- **M-I :** Linear Algebra
- **M-II :** Graph Theory
- **S-I :** Methods of Applied Statistics
- **S-II :** Continuous Probability Distributi and Testing of Hypothesis