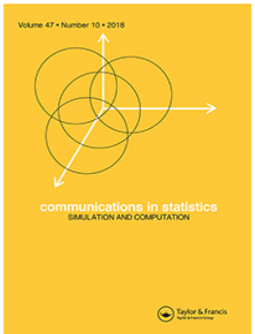

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Sample based side sensitive group runs control chart to detect shifts in the process mean

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ABSTRACT
In this article, we propose side sensitive group runs based univariate control chart using sample based 'transition probability matrix' (tpm) namely 'Sample based Side Sensitive Group Runs' (S-SSGR) control chart. The zero state ATS performance of the S-SSGR chart and the CRL based 'Side Sensitive Group Runs' (SSGR) chart is exactly same but better than the 'Side Sensitive Synthetic' (SSS) chart and the 'Group Runs' (GR) Chart. Also the steady state ATS performance of the S-SSGR chart is better as compared to the GR and the SSGR charts.

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Synthetic control chart; Side sensitive group runs control chart; Transition probability matrix; Zero state; Steady state; Conforming run length

1. Introduction

To monitor the process mean, Albin, Kang, and Shea (1997), Ncube (1990) and Shamma and Shamma (1992) developed various combinations of charts. Further, using 'Average Run Length' (ARL) model, Wu and Spedding (2000) introduced the synthetic control chart to detect shifts in the process mean. This chart is a combination of the CRL chart and the Shewhart's $\overline{X}$ chart. In zero state, they have illustrated that the synthetic chart performs better than the CRL chart and the Shewhart's $\overline{X}$ chart for small to moderate shifts in the process mean. They have not studied the steady state performance of the synthetic chart. Davis and Woodall (2002) highlighted an important aspect of side sensitivity and have illustrated that it can be used to improve the performance of the synthetic control chart. In side sensitivity, a type of the shift along with the value of the CRL is taken into consideration. They defined the runs rule for 'Side Sensitive Synthetic' (SSS) control chart as 'Declare the process as out of control, if two out of $(L+1)$ sample means fall beyond the control limits on the same side of the target value'. Here L is the lower control limit of the SSS control chart. In the runs rule of SSS control chart, they have not used the term 'successive'. While defining the above runs rule, they assumed that, at time zero, there is a nonconforming sample having a shift of both the sides of the target value. Such type of assumption is known as a head start. Meaning of this rule is "if the 1st nonconforming sample being observed and if the sample mean is above/below 'Upper Control Limit' (UCL)/'Lower Control Limit' (LCL), it is assumed that, at time zero, sample mean is above UCL/below LCL. Though both the

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statements about the assumption have the same meaning, latter is much easy to understand. As an illustration, Davis and Woodall (2002), provide the non-zero elements of the transition probability matrix for L = 2. Here, from the state + – means the current sample mean is below LCL and previous sample mean above UCL. If the next sample mean is above UCL (or below LCL) then the corresponding state will be – + (or – –). Looking to the tpm of the SSS chart proposed by Davis and Woodall (2002), the $(6, 7)^{th}$ and $(6, 10)^{th}$ cell probabilities are 0 and LO + HI respectively. Note that the respective cell probabilities should be HI and LO.

Gadre and Ratnaparkhe (2006) developed CRL based SSS control chart. They have corrected the runs rule as 'Declare the process as out of control, if two out of (L + 1) successive sample means fall beyond the control limits on the same side of the target value'. They pointed out the runs rule introduced by Davis and Woodall (2002) with the help of the following illustration. Let L (lower control limit) of the SSS chart introduced by Davis and Woodall (2002) be 8 and further assume that the first nine samples are conforming to the quality and the 10^{th}, 12^{th} and 15^{th} sample is nonconforming with 10^{th} and 15^{th} sample showing the shift to the same side, whereas 12^{th} sample showing the shift to the opposite side. In this case the operation of SSS chart defined by Davis and Woodall (2002) gives a signal, but as the successive nonconforming sample means are falling on the opposite side of the target value, a side sensitive synthetic control chart should not signal. These drawbacks are corrected in CRL based SSS chart proposed by Gadre and Ratnaparkhe (2006).

Moreover, Davis and Woodall (2002) have not studied the steady state ARL performance of the SSS chart. Gadre and Ratnaparkhe (2006) introduced a SSS control chart using CRL based tpm to detect shifts in the process mean. They have used ATS model, but not studied the steady state ATS performance of the chart.

In this article, we discuss the development of SSS chart introduced by Gadre and Ratnaparkhe (2006) along with the notations used. They used CRL based tpm to obtain design parameters of this chart. Gadre and Rattihalli (2004) proposed group runs control chart for detecting shifts in the process mean, which is a combination of Shewhart's $\overline{X}$ chart and an extended version of conforming run length chart.

Also Gadre and Rattihalli (2007) proposed the 'Side Sensitive Group Runs' (SSGR) control chart to detect small shifts in the process mean. These two charts are based on CRL based tpm. Here we develop the 'Sample based SSGR' (S-SSGR) chart. We studied the zero state and the steady state ATS performance of the S-SSGR chart and compared with that of the SSGR chart. Here, in zero state, ATS performance of the SSGR chart and that of the S-SSGR chart is same. In steady state, given the value of L, states of the SSGR chart are (4 L + 1) and those of the proposed S-SSGR chart are $(4L^2_{S-SSG} + 3L_{S-SSG} + 1)$. As the proposed chart has more precise states as compared to those of the SSGR chart, the steady state ATS performance shows that it is better as compared to that of the SSGR chart.

This article is organized as follows. In Sec. 2, a review of the Run Length based control charts is described brief. The notations used in the CRL based SSS chart, its operation, design and the ATS expression proposed by Gadre and Ratnaparkhe (2006) are described in Sec. 3. Also numerical examples give the design parameters and ATS_1 values of the CRL based SSS chart and the SSS chart in the same section. Section 4 includes the runs rule representation and operation of the proposed S-SSGR chart and the numerical illustrations, which compare the zero state ATS performance of the proposed chart with the SSS chart, the GR chart and the SSGR chart. Further steady state ATS performance of the S-SSGR chart is discussed in Sec. 5. Concluding remarks of the propose chart are given in the last section.

2. A review of the run length based control charts

2.1. Synthetic control chart

The 'Conforming Run Length' (CRL) chart proposed by Bourke (1991) can be used to detect a shift in the process level. Wu and Spedding (2000) proposed the synthetic control chart for detecting small shifts in the process mean by combining the Shewhart's $\overline{X}$ chart and the CRL chart. They defined Y_r (the r^{th} sample based CRL) as the number of conforming sample inspected between $(r-1)^{th}$ and r^{th} nonconforming sample, including the r^{th} nonconforming sample. Synthetic chart declares the process as out of control when for some $r \geq 1$, $Y_r \leq L_s$ for the first time. Here L_s is the lower control limit of the synthetic chart.

2.2. 'Side sensitive synthetic' (SSS) control chart

Davis and Woodall (2002) highlighted important aspects of side sensitivity. They defined the rules for side sensitive synthetic control chart as 'Declare the process as out of control if two out of $(L_{ss} + 1)$ sample means fall outside the control limits on the same side of the target value'. Davis and Woodall (2002) illustrated that, in zero state, the ARL performance of SSS chart is better than that of the synthetic chart. They have not studied and compared the steady state performance of both the charts.

2.3. SSGR control chart

Gadre and Rattihalli (2007) developed a 'Side Sensitive Group Runs' (SSGR) control chart to detect shifts in the process mean. They find optimal values of the design parameters (n, k, L_{SG}) of the chart. The SSGR chart will signal, 'if $Y_1 \leq L_{SG}$ (the lower limit of the chart) or for some $r \geq 2$, $Y_r \leq L_{SG}$ and $Y_{r+1} \leq L_{SG}$ and the r^{th} nonconforming group and $(r+1)^{th}$ nonconforming group (if one such exists) are indicating a shifts on the same side of a target value μ_0 of the process mean'.

3. CRL based SSS control chart

Gadre and Ratnaparkhe (2006) developed a 'Side Sensitive Synthetic' (SSS) control chart to detect shifts in the process mean with corrected runs rule in SSS chart proposed by Davis and Woodall (2002). They found optimal values of the design parameters (n, k, L_{SS}) of the chart. While defining the SSS chart, they assumed the head start as mentioned in Davis and Woodall (2002). In the following, we enlist the notations required for the above chart.

3.1. Notations and terms

Following are some notations required for the SSS chart.

1. μ_0: In-control value of the process mean;
2. σ: The process variability;
3. ATS(δ): average number of units required by the SSS chart to detect a shift in process mean from μ_0 to $\mu_0 \pm \delta\sigma$;
4. δ_1: shift in the mean, the magnitude of which is considered large enough to seriously impair the quality of the product;
5. n: The sample size;
6. k: The coefficient used in the control limits of the sub-chart;
7. L_{SS}: Lower limit of the SSS control chart;
8. Y_r: The r^{th} ($r = 1, 2, \ldots$) value of the sample based CRL, when the groups are treated as units. i.e. it is the number of groups inspected between $(r\text{-}1)^{th}$ and r^{th} non-conformed group, including the r^{th} non-conformed group.
9. $ATS_0 = ATS(0)$ and $ATS_1 = ATS(\delta_1)$;
10. τ: the minimum required value of ATS_0.

3.2. Implementation of SSS chart

Following is the operation of the SSS chart.

1. Observe n items in succession constituting a sample.
2. Declare the sample as conformed (nonconforming) according as the sample mean falls within (outside) the limits $L_{\overline{X}/s} = (\mu_0 - k\sigma/\sqrt{n})$ and $U_{\overline{X}/s} = (\mu_0 + k\sigma/\sqrt{n})$ of the $\overline{X}$ sub chart.
3. Declare the process as out of control, if for some $r \geq 1$, $Y_r \leq L_{SS}$, (lower limit of the SSS control chart) for the first time and the related nonconforming sample means lie on the same side of the target value μ_0.

When the process goes out of control, stop the process, take corrective actions to set the target value and then restart the process. Below we describe the design of the SSS chart.

3.3. Design of the SSS chart

Values of the design parameters (n, k, L_{SS}) are computed by using the following ATS criterion.

$$\left.\begin{array}{c}\text{Minimize } ATS_1 \\ \text{subject to the constraint} \\ ATS_0 \geq t.\end{array}\right\} \tag{1}$$

If N is the number of nonconforming samples observed before declaring the process has gone out of control then under the assumption that the units produced are per unit time, ATS is the average number of units required to declare the process as out-of-control. Hence, ATS = n ARL. Where, ARL is $E(\sum_{r=1}^{N} Y_r)$.

$$ATS(\delta) = nE\left(\sum_{r=1}^{N} Y_r\right). \tag{2}$$

Using Wald's identity (Zacks (1971) p. 202), from (2), ATS(δ) = n E(Y_r)E(N).

Let P be the probability that the sample being nonconforming when the process mean is shifted from μ_0 to $\mu_0 \pm \delta\sigma$. If it is assumed that X has normal distribution then,

$$\mathrm{P} = \mathrm{P}\,(\delta) = 1 - \mathrm{P}\left\{L_{\overline{X}/S} < \overline{X} < U_{\overline{X}/S} | \overline{X} \sim \mathrm{N}\left(\mu_0 + \delta\sigma, \sigma/\sqrt{\mathrm{n}}\right)\right\}.$$

Thus, we have,

$$\mathrm{P} = 1 - \Phi\left(k - \delta\sqrt{n}\right) + \Phi\left(-k - \delta\sqrt{n}\right). \tag{3}$$

Notice that, Y_r (r = 1, 2, …) are independent and identically distributed (i.i.d.) geometric random variables with mean (1/P). Therefore,

$$\mathrm{ATS}(\delta) = \frac{n}{\mathrm{P}} E(\mathrm{N}). \tag{4}$$

3.4. Derivation of the ATS expression by using markov chain representation of the SSS chart and the CRL based tpm

Davis and Woodall (2002) have discussed the Markov chain representation of the SSS control chart to identify shifts in the process mean using sample based tpm. Gadre and Ratnaparkhe (2006) discussed the the Markov chain representation of the SSS chart using CRL based tpm. Following are the notations to write a tpm of the Markov chain representation of the SSS chart.

1. $A = P(Y_r \leq L_{SS}) = 1 - (1-P)^{L_{SS}}$
2. $\alpha = (1-\phi(k - \delta\sqrt{n}))/P$
3. $\overline{m} = \{Y > L_{SS} \text{ and } \overline{X} > U_{\overline{X}/S}\}$
4. $\underline{m} = \{Y > L_{SS} \text{ and } \overline{X} < L_{\overline{X}/S}\}$
5. $\overline{l} = \{Y \leq L_{SS} \text{ and } \overline{X} > U_{\overline{X}/S}\}$
6. $\underline{l} = \{Y \leq L_{SS} \text{ and } \overline{X} < L_{\overline{X}/S}\}$
7. $\underline{\overline{m}} = \{Y_0 \leq L_{SS}, \overline{X} < L_{\overline{X}/S} \text{ and } \overline{X} > U_{\overline{X}/S}\}$, This is an artificial situation, which is consider as head start.

Then, the tpm corresponding to the SSS chart can be given as below.

$$\begin{array}{c} \\ \overline{\overline{m}} \\ \overline{m} \\ \underline{m} \\ \bar{l} \\ \underline{l} \\ \text{Signal} \end{array}\begin{array}{c} \begin{array}{cccccc} \overline{\overline{m}} & \overline{m} & \underline{m} & \bar{l} & \underline{l} & \text{Signal} \end{array} \\ \begin{bmatrix} 0 & \alpha(1-A) & (1-\alpha)(1-A) & 0 & 0 & A \\ 0 & \alpha(1-A) & (1-\alpha)(1-A) & 0 & (1-\alpha)A & \alpha A \\ 0 & \alpha(1-A) & (1-\alpha)(1-A) & \alpha A & 0 & (1-\alpha)A \\ 0 & \alpha(1-A) & (1-\alpha)(1-A) & 0 & (1-\alpha)A & \alpha A \\ 0 & \alpha(1-A) & (1-\alpha)(1-A) & \alpha A & 0 & (1-\alpha)A \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array} \tag{5}$$

In this case, the initial state is $\overline{\overline{m}}$. Hence, taking the sum of the elements of the first row of $(I - R)^{-1}$, E(N) can be obtained. Using 'Mathematica' software, E(N) is

$$E(N) = \frac{1 + A^2\alpha(\alpha-1)}{A\{1 + \alpha(1-\alpha)(A-2)\}} \tag{6}$$

and as in Eq. (4), ATS(δ) is

$$ATS(\delta) = \frac{n}{P}\frac{1 + A^2\alpha(\alpha-1)}{A\{1 - \alpha(1-\alpha)(A-2)\}}. \tag{7}$$

3.5. Numerical illustrations

It is preferred to use ATS model as compared to the ARL model. We consider sets of input parameters to compare the performance of CRL based SSS chart with the SSS (Davis and Woodall) chart.

Example 1: For the input parameters $(\mu_0, \sigma, \delta_1, \tau) = (0, 1, 0.2, 10000)$, values of the design parameters (n, k, L_{SS}) of the SSS charts along with respective ATS_1 values are given in Table 1.

Entries in the above table show that, ATS_1 values of all the above SSS charts for mean are exactly same.

Example 2: For $L > 3$, $\mu_0 = 0$, $\sigma = 1$, $\delta_1 = 0.2$ and $\tau = 10000$ the respective ATS_1 values are given in Table 2.

For $L > 3$, the following are the observations.

Table 1. Design parameters and ATS_1 values of the two charts.

Control Chart	n	k	L	ATS_1
CRL based SSS Chart	103	1.743	3	178.3909
SSS Chart	103	1.743	3	178.3909

Table 2. Design parameters and ATS_1 values of the two charts for $L > 3$.

L	Control Chart	n	k	ATS_1
4	CRL based SSS Chart	**100**	1.814	**180.2083**
	SSS Chart	127	1.935	207.1784
5	CRL based SSS Chart	**95**	1.874	**183.4146**
	SSS Chart	122	1.961	206.1643

1. ATS_1 values of the CRL based SSS chart are smaller than that of the SSS chart.
2. The sample size n of the SSS chart is greater than that of the sample sizes of the CRL based SSS chart.

As Davis and Woodall (2002) and Gadre and Ratnaparkhe (2006) have not studied the steady state performance of the respective charts, it is not essential to compare the steady state ATS performance of these two charts.

4. Sample based S-SSGR control chart

In zero state case, for CRL based SSGR chart, tpm is of fixed order (10x10) not depending on L_{SG}, but in case of S-SSGR chart, the order of tpm is depending on $L_{S\text{-}SG}$. In this section, we propose the notations and the exact runs rule for non absorbing states of the S-SSGR chart. The number of non absorbing states and state of signal for this chart is $(5\,L_{S\text{-}SSG} + 3)$.

We enlist the notations for the S-SSGR chart as follows:

4.1. Notations and terms

1. $P_c = P(\overline{X} \in (LCL_{\overline{X}}, UCL_{\overline{X}}))$
2. $P_u = P(\overline{X} > UCL_{\overline{X}})$
3. $P_l = P(\overline{X} < LCL_{\overline{X}})$
4. '+' indicates a non conforming sample whose sample mean is above $UCL_{\overline{X}}$
5. '–' indicates a non conforming sample whose sample mean is below $LCL_{\overline{X}}$
6. '0' indicates a sample being conforming
7. '±' indicates that at time zero, a sample being nonconforming such that if the first non conforming sample whose sample mean is above $UCL_{\overline{X}}$ then at time zero, the sample mean is above $UCL_{\overline{X}}$ and if the first non conforming sample whose sample mean is below $LCL_{\overline{X}}$ then at time zero, the sample mean is below $LCL_{\overline{X}}$. It is a imaginary head start.

4.2. Markov chain representation of the S-SSGR chart

For some $r > 1$, if $Y_r \le L_{S\text{-}SSG}$ with the corresponding sample mean of the nonconforming sample is above/below UCL/LCL and if $Y_{r\text{-}1} > L_{S\text{-}SSG}$ with the sample mean of nonconforming sample at the right end is also above/below UCL/LCL is denoted by $\overline{1}$ /$\underline{1}$.

1. At time zero, $Y_0 \le L_{S\text{-}SSG}$ and non confirming sample on both side of target value (±) can be treated as a 'Head start' (No. of states = 1).
2. A sequence in (1) is followed by at most ($L_{S\text{-}SSG}$ - 1) 0's. (No. of states = $L_{S\text{-}SSG}$ - 1).
3. A sequence of at least $L_{S\text{-}SSG}$ 0's. (No. of states = 1).
4. Sequence in (3) is followed by +/- and further append by at most ($L_{S\text{-}SSG}$ - 1) 0's. (No. of states = $2\,L_{S\text{-}SSG}$).
5. $\overline{1}/\underline{1}$ is followed by at most ($L_{S\text{-}SSG}$ - 1) 0's (No. of states = $2\,L_{S\text{-}SSG}$)
6. 0+/0- (No. of states = 2)

7. Signal.

Therefore, $\mathbf{R_1}$ is a square matrix of order 1+ ($L_{S\text{-}SSG}$ - 1) + 1 + 2 $L_{S\text{-}SSG}$ + 2 $L_{S\text{-}SSG}$ + 2 = 5 $L_{S\text{-}SSG}$ +3.

For e.g. $L_{S\text{-}SSG}$ = 3, Total of non absorbing states = 18.

4.3. Operation of S-SSGR chart

Though the operations of SSGR and S-SSGR control charts are exactly same, it is preferred to explain the operation of the proposed chart. Following are the stepwise procedures.

1. Observe n items in succession constituting a group.
2. Declare the group as conformed (nonconforming) according as the group mean falls within (outside) the limits $L_{\overline{X}/s} = (\mu_0 - k\sigma/\sqrt{n})$ and $U_{\overline{X}/s} = (\mu_0 + k\sigma/\sqrt{n})$ of the $\overline{X}$ sub chart.
3. Declare the process as out of control, if $Y_1 \le L_{S\text{-}SSG}$, (lower limit of the S-SSGR control chart) or for some $r > 1$, $Y_r \le L_{S\text{-}SSG}$, $Y_{r+1} \le L_{S\text{-}SSG}$ for the first time and the related nonconforming group means lie on the same side of the target value μ_0.

When the process goes out of control, stop the process, take corrective actions to set the target value and then restart the process.

A Macro in Mat-Lab is developed to obtain the design parameters of the S-SSGR chart for given input parameters and is used for the following illustrations.

4.4. Numerical illustrations

It is preferred to use ATS model as compared to ARL model. We consider the sets of input parameters to compare the performance of the S-SSGR chart with the GR and the SSGR charts based on CRL proposed by Gadre & Rattihalli (2004) and Gadre and Ratnaparkhe (2006) respectively.

Example 3: For the input parameters ($\mu_0 = 0$, $\sigma = 1$, δ_1, τ), we have considered 4 values of δ_1 and 3 values of τ and obtained values of the design parameters for the three charts along with respective ATS_1 values are given in Table 3.

All entries in the above table are exactly same for the SSGR and that of the S-SSGR charts for mean and gives better zero state ATS performance than those of the SSS chart and GR chart.

5. Steady state ATS performance of the S-SSGR chart

Davis and Woodall (2002), proposed runs rule for the synthetic control chart for the steady state performance. Gadre and Rattihalli (2004), Gadre and Ratnaparkhe (2006) considered the steady state performance of the GR and the SSGR charts for detecting shifts in the process mean. But in steady state performance of the SSGR chart, though

Table 3. Design parameters and ATS_1 values of the three charts.

			SSS Chart				GR Chart				SSGR Chart				S-SSGR Chart			
Ex.	δ_1	τ	n	k	L	ATS1	n	k	L	ATS_1	n	k	L	ATS_1	n	k	L	ATS_1
1	0.2	2000	72	1.345	2	129.7	63	1.457	4	123.58	59	1.295	3	113.05	57	1.302	3	113.16
2	0.3	2000	37	1.609	3	68.01	37	1.477	3	64.15	33	1.407	3	58.85	33	1.407	3	58.85
3	0.4	2000	24	1.711	3	42.90	23	1.556	3	39.74	21	1.490	3	36.57	21	1.490	3	36.57
4	0.5	2000	18	1.777	3	29.81	16	1.63	3	27.21	16	1.549	3	25.13	16	1.549	3	25.13
5	0.2	5000	93	1.513	2	158.81	83	1.497	3	147	75	1.425	3	134.97	75	1.425	3	134.97
6	0.3	5000	48	1.762	3	81.20	44	1.614	3	74.46	39	1.542	3	68.66	39	1.542	3	68.66
7	0.4	5000	30	1.866	3	50.43	28	1.692	3	45.51	25	1.618	3	42.09	25	1.618	3	42.09
8	0.5	5000	21	1.943	3	34.68	19	1.756	3	30.912	17	1.682	3	28.69	17	1.682	3	28.69
9	0.2	10000	103	1.743	3	178.39	98	1.59	3	165	89	1.52	3	151.78	89	1.52	3	151.78
10	0.3	10000	55	1.885	3	91.40	50	1.711	3	82.216	46	1.632	3	76.11	46	1.632	3	76.11
11	0.4	10000	35	1.982	3	56.29	31	1.789	3	49.85	28	1.713	3	46.30	28	1.713	3	46.30
12	0.5	10000	24	2.059	3	38.44	21	1.851	3	33.712	19	1.774	3	31.37	19	1.774	3	31.37

Table 4. The initial states corresponding to S-SSGR sample base chart when $L_{S\text{-}SSG} = 2$.

Sr. No.	State	Sr. No.	State
1	$\underline{\overline{1}}$	13	$\underline{1}\ 0$
2	$\underline{\overline{1}}\ 0$	14	$\overline{1}$
3	$\underline{\overline{1}}\ \overline{1}$	15	$\overline{1}\ 0$
4	$\underline{\overline{1}}\ \overline{1}\ 0$	16	$\underline{1}\ \underline{1}$
5	$\underline{\overline{1}}\ 0\ \overline{1}$	17	$\underline{1}\ \underline{1}\ 0$
6	$\underline{\overline{1}}\ 0\ \overline{1}\ 0$	18	$\underline{1}\ 0\ \underline{1}$
7	$\underline{\overline{1}}\ \underline{1}$	19	$\underline{1}\ 0\underline{1}\ 0$
8	$\underline{\overline{1}}\ \underline{1}\ 0$	20	$\overline{1}\ \overline{1}$
9	$\underline{\overline{1}}\ 0\underline{1}$	21	$\overline{1}\ \overline{1}\ 0$
10	$\underline{\overline{1}}\ 0\underline{1}\ 0$	22	$\overline{1}\ 0\ \overline{1}$
11	00	23	$\overline{1}\ 0\ \overline{1}\ 0$
12	$\underline{1}$	24	Signal

the process is running smoothly for a long time, they assumed that before monitoring the product (i.e. at time zero), $Y_0 \le L_{S\text{-}SSG}$ and nonconforming sample on both sides of target value. This assumption is referred to as a 'Head Start' which is not desirable to use for steady state performance. Here, we propose new runs rule for the S-SSGR chart which are distinct from the SSGR chat for the steady state performance.

5.1. The Markov chain representation of the S-SSGR chart

The notations P_c, P_u, P_l, $+$, $-$, 0 and $\pm$ which are define in earlier sub-section 4.1 proposed by Davis and Woodall (2002).

When the process is running smoothly for a long time, it is preferred to assume that at time zero, $Y_0 > L_{SSG}$ and non confirming sample on both sides of target value. The corresponding notation is $\underline{\overline{1}}$. Further the notations $\overline{1}$, $\underline{1}$ and 0 are defined in sub-section 4.2. We propose the Markov chain representation of the sample based tpm is as follows:

1. $\underline{\overline{1}}$ is followed by at most ($L_{S\text{-}SSG}$ - 1) 0's (No. of states $= L_{S\text{-}SSG}$).
2. A sequence in (1) is followed by $\overline{1}/\underline{1}$ further append by at most ($L_{S\text{-}SSG}$ -1) 0's. (No. of states $= 2\,L_{S\text{-}SSG}{}^2$).
3. A sequence of at least $L_{S\text{-}SSG}$ 0's. (No. of states $= 1$).
4. $\underline{1}/\overline{1}$ is followed by at most ($L_{S\text{-}SSG}$ - 1) 0's. (No. of states $= 2\,L_{S\text{-}SSG}$).

5. A sequence in (4) is followed by $\underline{1}/\overline{1}$ and is further appended by at most at most ($L_{S\text{-}SSG}$ -1) 0's (No. of states $= 2\,L_{S\text{-}SSG}{}^{2}$)

Therefore, $\mathbf{R_1}$ is a square matrix of order $2 + 3\,L_{S\text{-}SSG} + 4\,L_{S\text{-}SSG}{}^{2}$.

For e.g. $L_{S\text{-}SSG} = 2$, total of non absorbing states $= 24$ and the states of the S-SSGR chart are given in Table 4.

The related one step tpm is given below.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1	0	P_c	P_u	0	0	0	P_1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	P_u	0	0	0	P_1	0	P_c	0	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	P_c	0	0	0	0	0	0	0	P_1	0	0	0	0	0	0	0	0	0	0	0	P_u
4	0	0	0	0	0	0	0	0	0	0	P_c	P_1	0	0	0	0	0	0	0	0	0	0	0	P_u
5	0	0	0	0	0	P_c	0	0	0	0	0	P_1	0	0	0	0	0	0	0	0	0	0	0	P_u
6	0	0	0	0	0	0	0	0	0	0	P_c	P_1	0	0	0	0	0	0	0	0	0	0	0	P_u
7	0	0	0	0	0	0	0	P_c	0	0	0	0	0	P_u	0	0	0	0	0	0	0	0	0	P_1
8	0	0	0	0	0	0	0	0	0	0	P_c	0	0	P_u	0	0	0	0	0	0	0	0	0	P_1
9	0	0	0	0	0	0	0	0	0	P_c	0	0	0	P_u	0	0	0	0	0	0	0	0	0	P_1
10	0	0	0	0	0	0	0	0	0	0	P_c	0	0	P_u	0	0	0	0	0	0	0	0	0	P_1
11	0	0	0	0	0	0	0	0	0	0	P_c	P_1	0	P_u	0	0	0	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0	0	0	0	0	0	0	P_u	0	P_1	0	0	0	0	0	0	0	0
13	0	0	0	0	0	0	0	0	0	0	P_c	0	0	P_u	0	0	0	P_1	0	0	0	0	0	0
14	0	0	0	0	0	0	0	0	0	0	0	P_1	P_c	0	P_c	0	0	0	0	P_u	0	0	0	0
15	0	0	0	0	0	0	0	0	0	0	P_c	P_1	0	0	0	0	0	0	0	0	0	P_u	0	0
16	0	0	0	0	0	0	0	0	0	0	0	0	0	P_u	0	0	P_c	0	0	0	0	0	0	P_1
17	0	0	0	0	0	0	0	0	0	0	P_c	0	0	P_u	0	0	0	0	0	0	0	0	0	P_1
18	0	0	0	0	0	0	0	0	0	0	0	0	0	P_u	0	0	0	0	P_c	0	0	0	0	P_1
19	0	0	0	0	0	0	0	0	0	0	P_c	0	0	P_u	0	0	0	0	0	0	0	0	0	P_1
20	0	0	0	0	0	0	0	0	0	0	0	P_1	0	0	0	0	0	0	0	0	P_c	0	0	P_u
21	0	0	0	0	0	0	0	0	0	0	P_c	P_1	0	0	0	0	0	0	0	0	0	0	0	P_u
22	0	0	0	0	0	0	0	0	0	0	0	P_1	0	0	0	0	0	0	0	0	0	0	P_c	P_u
23	0	0	0	0	0	0	0	0	0	0	P_c	P_1	0	0	0	0	0	0	0	0	0	0	0	P_u
24	0	0	0	0	0	0	0	0	0	0	0	P_1	0	0	0	0	0	0	0	0	0	0	0	1

For the general value of $L_{S\text{-}SSG}$, the matrix $\mathbf{R}_1$, (eliminating the last row and the last column of the tpm) has the following states.

Let $\boldsymbol{\pi}$ be a 1 x $(4\,L_{S\text{-}SSG}{}^{2} + 3\,L_{S\text{-}SSG} + 2)$ row vector corresponding to the stationary probability distribution that the Markov chain will be in each of the non absorbing states, which is conditioned on no signal. Then the steady state ARL of the S-SSGR sample base chart can be obtained by $\pi\ (I\text{-}\ R_1)^{-1}\underline{1}$ and the SSATS is the product of $n_{S\text{-}SSG}$ and the steady state ARL.

The performances of any two charts should be compared by making the in-control Steady State ATS (SSATS(0)) of the two charts the same. The Steady State ATS value for the shift δ is denoted by SSATS(δ). Computation of the adjusted steady state ATS (Adj(SSATS)) of Chart II with respect to Chart I is defined as below.

$$\left[\mathrm{Adj(SSATS}(\delta)\right]_{\mathrm{II}} = \ \left\{[\mathrm{SSATS}(\delta)]_{\mathrm{II}}/\ [\mathrm{SSATS}(0)]_{\mathrm{II}}\right\}\left\{\mathrm{SSATS}(0)_{\mathrm{I}}\right\}. \tag{8}$$

Example 3 (Cont.):

Table 5. Values of steady state (ATS) corresponding to various values of δ.

	GR Chart		SSGR Chart		S-SSGR Chart	
δ	SSATS	Adj. SSATS	SSATS	Adj. SSATS	SSATS	Adj. SSATS
0	13531	**10000**	15542.00	**10000**	26283.00	**10000**
0.1	1580.59	**1168.13**	1312.80	**844.68**	1351.40	**514.17**
0.2	306.36	**226.41**	308.19	**198.29**	324.09	**123.31**
0.26	204.64	**151.23**	219.50	**141.23**	233.27	**88.75**
0.3	178.62	**132.01**	195.54	**125.81**	208.54	**79.34**
0.4	158.91	**117.44**	176.06	**113.28**	188.35	**71.66**
0.5	156.89	**115.95**	173.69	**111.76**	185.89	**70.73**

Table 5 gives the ‘Steady State ATS’ (SSATS) and the ‘Adjusted Steady State ATS’ (Adj. SSATS) values for design parameters corresponding to Example 3 for all the three charts.

From Table 5, for all δ values, when $\tau = 10000$,

$(\text{Adj. SSATS})_{\text{S-SSGR}} \leq (\text{Adj. SSATS})_{\text{SSGR}} \leq (\text{Adj. SSATS})_{\text{GR}}$

This indicates the steady state ATS performance of the S-SSGR chart is better as compared to the other two charts.

6. Conclusion

The zero state ATS performance of the SSGR chart and that of the S-SSGR chart is exactly same, but better than the SSS and the GR Charts. Also the steady state ATS performance of the S-SSGR chart is better as compared to the GR and the SSGR charts.

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