COMPARATIVE STUDY OF SELF RELOCATING DESIGN AND TYPE II CENSORING UNDER EXPONENTIAL DISTRIBUTION

Chandrashekhar P. Swami Department of Statistics, Shri Jagdish Prasad Jhabarmal Tibrewala University, Jhunjhunu, Rajasthan- India - shekhars793@gmail.com
Vaishali V. Patil Department of Statistics, Tuljaram Chaturchand College of Arts, Science and Commerce, Baramati, Dist-Pune, Maharashtra- India - vaishutcc@gmail.com

Abstract: Both the manufacturing and service industries try to improve the quality and reliability of their products. Reliability is the probability that a product or equipment will perform satisfactorily for a given time under normal conditions of use. In a similar way, more than one treatment is used to treat a specific disease in the medical field. There is a different level of reliability for each treatment. In this study, we considered radiotherapy and the combination of radiotherapy and chemotherapy used in patients with head and neck cancer. The lifetime of the data under study follows an exponential distribution. In this study, we considered SRD and Type II censoring designs to determine the estimates of the parameters of the distribution under study: survival function, hazard function, and their standard errors. We also study design optimality criteria by performing studies of these designs using simulation techniques and real data. We found that self-relocating design is better than Type II censoring on the basis of optimality criteria. This study reveals that the probability of patients surviving for six or one year is higher if radiotherapy (RT) is used instead of a combination of radiotherapy and chemotherapy (RT+CT).

Keywords: Self Relocating Design, Reliability, Type-II Censoring, Exponential Distribution, Survival function, Hazard function

INTRODUCTION:

Reliability and survival analysis are the most popular and powerful techniques in statistics that deal with life-time data. In many industries, reliability analysis is used to estimate and predict the successful functioning or performance of products. The consumer has started to demand products with acceptable quality and reliability at a reasonable price. This guarantee of the product mostly depends on the product's reliability. On a similar line, survival analysis is used for analyze the time until the occurrence of an event like death, disease, recovery, or other experience of interest. Distinct products, system components, human lifetime, and other living things all have quite different patterns of survival. As a result, various failure time distributions are available to describe the variability present in the current data, Karim and Islam, (2019).

The experiment of testing the lifetime of the items in the experiment or the survival of an individual under study is difficult due to its cost and time constraints. The censorship scheme provides the solution to this problem. There are various censoring schemes available, of which the most popular is Type-II censoring. Srivastava (1986) introduced a new class of design called "self-relocating design" (SRD). The SRD is an alternative to current censoring methods that collects the performance of the item or individual under study on a variety of brands, presumably in a variety of settings. Recently, Shanubhogue and Raykundaliya (2015b) did a comparative study of self-relocating design (SRD) and Type-II censoring design. Exponential distribution is the more popular distribution under the lifetime study and A.A. Dharmadhikari et al. (2000) have considered two-way classification under the multiplicative model when the life time of the data follows an exponential distribution. Amita Dharmadhikari (2002) considered two different factors, the "brand of units" and the "environment," to test the main and interaction effects of the two-factor design. In all of these studies, reliability studies were carried out for the industrial products.

As we know now a days several treatments are available on the same disease. Hence the obvious question arises which treatment is used to treat the patient. To study the performance of each treatment one can, study by the type II censoring method or self-relocating design. Hence, on the similar line in this study, we have considered these procedures on secondary data. The data is considered from the medical field. There are two datasets, the data set-I represent the survival times of a group of patients suffering from Head and Neck cancer disease and treated using a radiotherapy (RT)and data set-II represent the survival times of a group of patients suffering from Head and Neck cancer disease and treated using a combination of radiotherapy chemotherapy (RT+CT), reported by Efron B (1988).

In Sections 2 and 3, we provided a brief description of the Type II censoring design and the self-relocated design, respectively. We covered the probability density function, the survival function, and the exponential distribution's hazard rate in Section 4. Additionally, we create likelihood equations for Type II and SRD21E censored data. In Section 5, we construct the equations for the asymptotic variance-covariance matrix and the maximum likelihood estimators of parameters for the SRD21E and Type II censoring designs. Additionally, the tables of ML estimates and their asymptotic standard errors, estimations of reliability and hazard rates, and their mean square error at a defined time point are provided in Section 6. These calculations were simulated using the Monte-Carlo simulation approach. For SRD and type II censoring schemes, we simulate the design optimality criterion in Section 7. Section 8 provides a few concluding comments.

MATERIALS & METHODS

Type II censoring Design

If experiment is more time consuming and items are costly then we can't put large number of items on test till all of them fail. To overcome this difficulty many censored life testing plans have been proposed. Many censoring techniques, such as Type-I censoring, Type-II censoring, hybrid censoring, and Type-II progressive censoring, are explored in the statistical literature to determine reliability. The two most frequent censoring processes are referred to as Type-I censoring and Type-II censoring schemes. These censorship systems are summarized as follows: Assume that n units are being observed in a specific life-testing experiment. The experiment continues up to a pre-specified time T (say) in the traditional Type-I censoring approach. The traditional Type-II censoring strategy requires the experiment to continue until G* failures occur out of n units in the experiment. In type II censoring design, we put two types of systems simultaneously on test in which for each system we start with u units and continue lifetime testing experiment until fix number of failures say G* are observed. So total number of items placed on experiment is mu, in this scheme we store failure time of each failed component in the variable t_{gi}, where g = 1, 2, ..., G* and i = 1, 2. Thus during the experiment, for type II censoring we record the data in (u, G*, t_{gi}); where, u stands for total number of units under lifetime testing experiment, G* is prespecified number of failures of the units and t_{gi} is time until i^{th} item gets failed.

Self-Relocated Design

The SRD is an alternative to current censoring methods that collect performance data on a variety of brands, presumably in a variety of settings. In the traditional type I and type II censorship studies, one must run m distinct experiments for each brand. If m different brands are to be tested under study in n different environmental conditions, then a total of mn number of independent experiments needs to be conducted. Srivastava suggested SRDijD notation to SRD model. In SRDijD design, all m brands are to be tested jointly under all n environmental conditions; failed units are replaced by new units of the same brand, here i denotes whether units added (i = 1) or removed (i = 2) from the system of experiment after each occurrence of failure of item; j denotes added/removed number of units are either equal in each failure (say 1) or it is random (say 2) and D stands for lifetime distribution under study. Thus, the experiments having data among themselves on the number of units to be tested for a specific combination of brand and environmental conditions would depend on how failures have occurred in other experiments. Srivastava (1986) has developed one-way analysis of SRDllE and Srivastava (1987 and 1989), has developed one-way analysis of SRD21W, in which 'brand of units' was considered as the source of variation. In SRDllE and SRD21W model, E and W means Srivastava has considered exponential and Weibull distribution under study. Dharmadhikari, et al. (2000) studied for two-way classification of SRDllE for RBD setup where 'brand of units' and 'environment in which unit works' was considered to be source of variation. Shanubhogue and Raykundaliya (2015b) has studied

SRD21model in one way classification with considering the life time data follows Generalized Exponential Distribution.

In this study, SRD21E was used for a one-way classification with unit brand as the source of variation. Consider m systems from various brands that were created for the same objective. Indicate the 1, 2, ..., m brands. Assume that systems from every brand are put through a life testing experiment in this same setting. When a system of a given brand fails, the failure time is reported together with the system in which failure has occurred. The number of systems from each brand in the life testing experiment is also kept consistent at all times by randomly dropping one system from each of the other brands at the time of failure. The experiment is carried out until there are G* failures. We see that G* is roughly equal to u. This method increases accuracy and decreases overall predicted experiment time for comparative experiments in the reliability field. The G failure times are denoted by t_1, t_2, ..., t_G and their corresponding type of system which failed at various time are recorded as i_1 , i_2,....i_{G*}. The label of the experiment those have failed $j'_{1i_1}, j'_{2i_2}, \dots\ j'_{Gi_{G*}}$. 'This design is Self-Relocating in a sense that the items being tested at a particular time depend upon the development of the (life) test itself.' Shanubhogue and Raykundaliya (2015)

Construction of Likelihood function and Maximum Likelihood Estimation

The scale family of distributions plays a vital role in lifetime data analysis. In reliability theory the most popular lifetime distribution is exponential distribution, because of its simplicity and mathematical feasibility. Along with this exponential distribution Rayleigh distribution, gamma distribution, Weibull distribution is such distribution which are widely used in reliability studies. Hence, we tried fit the data to exponential distribution. Consider a life time of item is random variable denoted by T and assumes to have exponential distribution, with probability distribution function

$$f(t;\ \beta) = \beta\ exp\{-\beta t\}\ ;\ t > 0, \beta > 0$$

Here β is scale parameter. The cumulative distribution function is

$$F(t,\beta)\ = \begin{cases} 0 & t < 0 \\ 1 - exp\{-\beta t\} & t\ \geq 0 \end{cases}$$

The corresponding reliability function is

$$S(t)\ =\ exp\{-\beta t\}$$

The corresponding Hazard Rate function is

$$r(t) = \beta$$

For Type II censoring Design:

The likelihood function of ith type of system for Type_II censoring design is as follows:

$$L_i(\beta,t) =\ \frac{u!}{(u-G^*)!}\prod_{g=1}^{G^*}\beta_i\ exp\{-\beta_i t_{gi}\}\ [exp\{-\beta_i t_{G^*i}\}]^{(u-G^*)}$$

Therefore, the likelihood function of complete system of experiment is as follows:

$$L = \prod_{i=1}^{m} L_i(\beta,t) =\ \prod_{i=1}^{m}\left\{\frac{u!}{(u-G^*)!}\prod_{g=1}^{G^*}\beta_i\ exp\{-\beta_i t_{gi}\}\ [exp\{-\beta_i t_{G^*i}\}]^{(u-G^*)}\right\}$$

We obtain maximum likelihood estimation of βi (i=1, 2) reliability function, hazard rate and observed Fisher information matrix under the type II censoring design. The log likelihood equation is

$$logL = \sum_{i=1}^{m} logL_i(\beta,t) =\ mlog\left(\frac{u!}{(u-G^*)!}\right) + \left[\sum_{i=1}^{m}\sum_{g=1}^{G^*}(log\beta_i - \beta_i t_{gi})\right] + (u-G^*)(-\beta_i t_{G^*i})$$

Differentiating $logL$ with respect toβ_i for i = 1, 2 we get the system of likelihood equation for Type II Censoring design as follows:

$$\frac{\partial logL}{\partial \beta_i} = \frac{G^*}{\beta_i} - \sum_{g=1}^{G^*} \mathrm{t_{gi}} - (u-G^*)\mathrm{t}_{G^*i} = 0 \qquad for\ \ i = 1, 2$$

Further solving the system of likelihood equations, we get the maximum likelihood estimator of $\underline{\beta} = (\beta_1, \beta_2)$ *as* $\underline{\hat{\beta}}$ is given by

$$\hat{\beta}_i = \frac{G^*}{\sum_{g=1}^{G^*} t_{gi} - (u - G^*) t_{G^*}} \quad for\ i = 1,2$$

The MLE of reliability $\bar{F}(t)$ and hazard rate r(t) for i[th] brand can be evaluated using invariance property of MLEs as $\hat{\bar{F}}_i(t_i) = \exp\{-\hat{\beta}_i t_i\}$; i = 1, 2 and $\hat{r}_i(t_i) = \hat{\beta}_i$; i = 1, 2. The expected Fisher information matrix will be used for constructing optimal censoring plans. The Observed fisher information matrix $I\left(\underline{\beta}\right)$ under the type II censoring design is used.

$$I\left(\underline{\beta}\right) = \begin{bmatrix} \frac{G_s}{{\beta_1}^2} & 0 \\ 0 & \frac{G_s}{{\beta_2}^2} \end{bmatrix}.$$

Hence variance covariance matrix of estimator $\underline{\hat{\beta}}$ is $I^{-1}\left(\underline{\beta}\right)$. We obtain three optimality criteria namely A- optimality, D-Optimality and E- Optimality.

For Self-Relocated Design (SRD21E):

The likelihood function of i[th] type of system for SRD21E is as follows:

$$L_i(\beta, t) = \frac{u!}{(u - G^*)!} \prod_{g=1}^{G^*} \beta_i\, exp\{-\beta_i t_{gi}\}\ [exp\{-\beta_i t_{G^*i}\}]^{(u-G^*)}$$

Therefore, the likelihood function of complete system of experiment is as follows:

$$L = \prod_{i=1}^{m} L_i(\beta, t) = \prod_{i=1}^{m} \left\{ \frac{u!}{(u - G^*)!} \prod_{g=1}^{G^*} \beta_i\, exp\{-\beta_i t_{gi}\}\ [exp\{-\beta_i t_{G^*i}\}]^{(u-G^*)} \right\}$$

We obtain maximum likelihood estimation of βi (i=1, 2) reliability function, hazard rate and observed Fisher information matrix under the type II censoring design. The log likelihood equation is

$$logL = \sum_{i=1}^{m} logL_i(\beta, t) = mlog\left(\frac{u!}{(u - G^*)!}\right) + \left[\sum_{i=1}^{m} \sum_{g=1}^{G^*} (log\beta_i - \beta_i t_{gi})\right] + (u - G^*)(-\beta_i t_{G^*i})$$

Differentiating *logL* with respect toβ_i for i = 1 , 2 we get the system of likelihood equation for Type II Censoring design as follows:

$$\frac{\partial logL}{\partial \beta_i} = \frac{G^*}{\beta_i} - \sum_{g=1}^{G^*} t_{gi} - (u - G^*) t_{G^*i} = 0 \qquad for\ \ i = 1,2$$

Further solving the system of likelihood equations we get the maximum likelihood estimator of $\underline{\beta} = (\beta_1, \beta_2)$ *as* $\underline{\hat{\beta}}$ is given by

$$\hat{\beta}_i = \frac{G^*}{\sum_{g=1}^{G^*} t_{gi} - (u - G^*) t_{G^*}} \quad for\ i = 1,2$$

The MLE of reliability $\bar{F}(t)$ and hazard rate r(t) for i[th] brand can be evaluated using invariance property of MLEs as $\hat{\bar{F}}_i(t_i) = \exp\{-\hat{\beta}_i t_i\}$; i = 1, 2 and $\hat{r}_i(t_i) = \hat{\beta}_i$; i = 1, 2. The expected Fisher information matrix will be used for constructing optimal censoring plans. The Observed fisher information matrix $I\left(\underline{\beta}\right)$ under the type II censoring design is used.

We obtain maximum likelihood estimation of β_i (i = 1, 2) reliability function, hazard rate and observed Fisher information matrix under the Self-Relocating Design.

The Likelihood equation for SRD design:

$\frac{\partial L}{\partial \beta_i} = \frac{\delta_i}{\beta_i} - \sum_{g=1}^{G^*} t_g - (u - G^*) t_{G^*} \quad i = 1,2;$

where, $\delta_i = \sum_{g=1}^{G^*} \delta_{ig}$; $i = 1, 2$ and

$$\delta_{ig} = \begin{cases} 1 & ;\ if\ type\ i\ system\ fails\ at\ time\ t_g \\ 0 & ;\ otherwise \end{cases}$$

G*: Number of systems failed, u: Total number of systems for each brand and t_g: g^{th} failure time of system.

The maximum likelihood estimation of $\hat{\beta}$ is given by

$$\hat{\beta}_i = \frac{\delta_i}{\sum_{g=1}^{G^*} t_{i_g} - (u - G)t_{G^*}}$$

The MLE of reliability $\bar{F}(t)$ and r(t) can be evaluated using invariance property of MLEs as

- $\hat{\bar{F}}_i(t_i) = \exp\{-\hat{\beta}_i t_i\}$; i = 1, 2
- $\hat{r}_i(t_i) = \hat{\beta}_i$; i = 1, 2

Observed Fisher Information Matrix under the Self Relocating Design:

$$V = \begin{bmatrix} \frac{\delta_1}{{\beta_1}^2} & 0 \\ 0 & \frac{\delta_2}{{\beta_2}^2} \end{bmatrix}$$

Optimality Criteria

Three optimality criteria namely A- Optimality, D-Optimality and E-Optimality we obtained for comparison of two designs. Here, we obtained these opticalities for Type II and SRD. These optimality criteria respectively defined as follows:

A- Optimality criterion: The A-optimality criteria minimizes the trace of the estimates of the covariance matrix of the model. The trace is nothing but the sum of diagonal values of a matrix. i.e. tr $(\hat{V}^{-1})$

D- Optimality criterion: The D-optimality criteria minimizes the determinant of the estimates of the covariance matrix of the model. The trace is nothing but the sum of diagonal values of a matrix. i.e. det $(\hat{V}^{-1})$

E- Optimality criterion: The E -Optimal criteria is to maximizes the minimum Eigen value of the information matrix $(\hat{V})$.

DATA ANALYSIS BY MONTE CARLO SIMULATION

Study under Type II Censoring Design

A Monte Carlo simulation study is conducted to compare the performance of the estimates. MLE are obtained for observation generated through the Type II censoring designs when numbers of two systems to be compared. All calculations are performed using R programming. We carryout simulation for the following parameter values

m =2, β_1=1.5, β_2=1.3, t=(0.1748, 0.2132), $\bar{F}(t)$ = (0.7693, 0.7579)

Table 1: Maximum Likelihood Estimate of Parameters, Reliabilityand Hazard Rate

u	G*		$\hat{\beta}_1$	$\hat{\beta}_2$	$\bar{F}_1(t)$	$\bar{F}_2(t)$	r1(t)	r2(t)
12	6	EV	1.7578	1.5629	0.7408	0.7347	1.7578	1.5629
		MSE	0.8266	0.7758	0.0104	0.0118	0.8266	0.7758
		SE	0.7176	0.6380	-	-	-	-
24	12	EV	1.6104	1.4327	0.7535	0.7533	1.6104	1.4327
		MSE	0.2533	0.2178	0.0049	0.0046	0.2533	0.2178
		SE	0.4649	0.4136	-	-	-	-
36	18	EV	1.5772	1.3791	0.7559	0.7576	1.5772	1.3791

		MSE	0.1627	0.1265	0.0032	0.0035	0.1627	0.1265
		SE	0.3718	0.3251	-	-	-	-
48	24	EV	1.5774	1.3751	0.7636	0.7630	1.5774	1.3751
		MSE	0.1205	0.0961	0.0021	0.0020	0.1205	0.0961
		SE	0.3220	0.2805	-	-	-	-

We observed that the means of MLE's for β_i for (i = 1, 2), reliability and hazard rate are very close to true value as number of systems put on test are increased. The average mean square errors are relatively small. Further we observed that estimates and mean square error are decreasing function of u of each system put on test.

Design Optimality Criteria: Average of variance-covariance matrices are computed for different simulated samples to evaluate trace of $\hat{V}^{-1}$, determinant of $\hat{V}^{-1}$and minimum eigen value of $\hat{V}$, to get A-optimality, D- Optimality and E-Optimality of the design respectively. All these calculations are performed using R programming. The results are summarized in table.

m=2, β1=1.5, β_2 =1.3, n=1000

Table 2: Optimality Criteria for Type II Censoring design

U	G*	A-optimality	D-optimality	E-optimality
12	12	0.9221	0.2097	1.9418
24	24	0.3872	0.0370	4.6270
36	36	0.2439	0.0146	7.2358
48	48	0.1825	0.0082	9.6456

From the above table we can see that for Type II Censoring A-optimality and D-optimality criterion decreases with number of systems put on test are increased. The E-optimality criterion is increases with increase in number of systems put on test are increased.

Monte Carlo Simulation Studyunder Self Relocating Design:

A Monte Carlo simulation study is conducted to compare the performance of the estimates. MLE,s are obtained for observation generated through the Self Relocating Design when numbers of two systems to be compared. All calculations are performed using R programming.We carryout simulation for the following parameter values:

m =2, β_1=1.5, β_2=1.3, t = (0.1748, 0.2132), $\bar{F}(t)$ = (0.7693, 0.7579)

Table 3: Maximum Likelihood Estimate of Parameters, Reliability and Hazard Rate

u	G		$\hat{\beta}_1$	$\hat{\beta}_2$	$\bar{F}_1(t)$	$\bar{F}_2(t)$	$r_1(t)$	$r_2(t)$
12	12	EV	1.6464	1.4322	0.7698	0.7401	1.6464	1.4322
		MSE	0.4672	0.4164	0.0077	0.0093	0.4672	0.4164
		SE	0.6223	0.6405	-	-	-	-

24	24	EV	1.5807	1.3344	0.7809	0.7462	1.5807	1.3344
		MSE	0.2045	0.1685	0.0035	0.0050	0.2045	0.1685
		SE	0.4766	0.3700	-	-	-	-
36	36	EV	1.5458	1.3545	0.7771	0.7502	1.5458	1.3545
		MSE	0.1349	0.1113	0.0024	0.0036	0.1349	0.1113
		SE	0.3457	0.3386	-	-	-	-
48	48	EV	1.5319	1.3299	0.7804	0.7517	1.5319	1.3299
		MSE	0.1020	0.0827	0.0019	0.0029	0.1020	0.0827
		SE	0.2708	0.3325	-	-	-	-

We observed that the means of MLE's for β_i for (i = 1, 2) reliability and hazard rate are very close to true value as number of systems put on test are increased. The average mean square errors are relatively small. Further we observed that estimates and mean square error are decreasing function of u of each system put on test.

Design Optimality Criteria for Self-Relocating Design: Average of variance -covariance matrices computed for different simulated samples to evaluate trace of $\hat{V}^{-1}$, determinant of $\hat{V}^{-1}$ and minimum eigen value of $\hat{V}^{-1}$, to get A-optimality, D-Optimality and E-Optimality of the design respectively. All calculations are performed using R programming. The results are summarized in table.

m = 2, $\beta_1 = 1.5$, $\beta_2 = 1.3$, n = 1000

Table 4: Optimality Criteria for Self-Relocating Design

U	G	A-optimality	D-optimality	E-optimality
12	12	0.5577	0.0770	3.2685
24	24	0.2996	0.0222	6.0908
36	36	0.2084	0.0108	8.9950
36	36	0.1566	0.0060	11.9236

From the above table we can see that for Self-Relocating Design is A-optimality and D-optimality criterion decreases with number of systems put on test are increased. The E-optimality criterion is increases with increase in number of systems put on test are increased.

Comparison between Type II Censoring and Self-Relocating Design on the basis A- optimality, D-Optimality and E-Optimality

In generalized type II censoring and self-relocating design, we studied the comparison of two brands. The form of distribution is considered as exponential distribution. Here we compare these two deigns with respect to optimality criteria.

Table5: Design Optimality Criteria

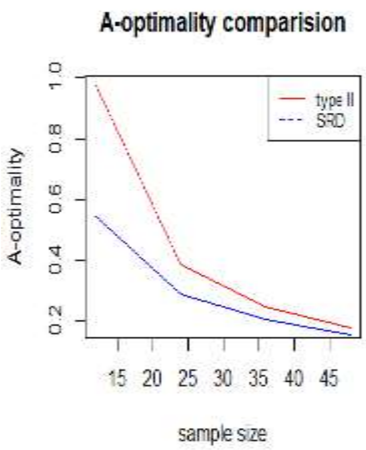
u	G^*	Types of Design	A-optimality	D-optimality	E-optimality
12	12	Type-II	0.9221	0.2097	1.9418
		SRD	0.5577	0.0770	3.2685
24	24	Type-II	0.3872	0.0370	4.6270

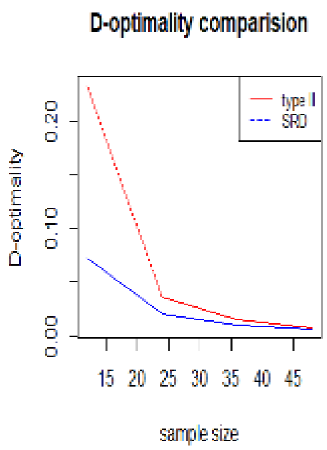
		SRD	0.2996	0.0222	6.0908
36	36	Type-II	0.2437	0.0146	7.2358
		SRD	0.2084	0.0108	8.9950
48	48	Type-II	0.1825	0.0082	9.6456
		SRD	0.1566	0.0061	11.9236

From the simulated data values of optimality criterion were estimated for SRD and Type-II censoring. Here we noticed that as number of systems u put on lifetime testing experiment is increased then the performance of Self Relocating Design always better than Type II Censoring with respect to A-optimality, D-optimality and for E-optimality criterion.

Graphical comparison of Type II Censoring and Self-Relocating Design on the basis A-optimality, D- Optimality and E-Optimality:

Optimum designs are a type of experimental design that is optimum in terms of some statistical criterion. The optimality of a design is determined by the statistical model and is measured against a statistical criterion that is connected to the estimator's variance-matrix. Specifying an adequate model and a suitable criteria function both need a grasp of statistical theory as well as actual experience in experiment design. When you wish to highlight certain model impacts, use an A-optimal design. The A-optimality requirements minimize the trace of the model's covariance matrix estimations. The trace is just the sum of a matrix's diagonal values. The D-optimality criteria minimizes the determinant of the model coefficient estimates' covariance matrix. As a result, D-optimality is concerned with exact estimations of the impacts. E-optimality is another design that maximizes the information matrix's smallest eigenvalue.





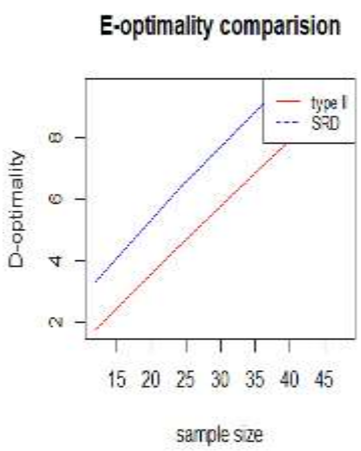


Figure1: A-Optimality Figure2: D-Optimality Figure3: E-Optimality

From above graphs we can observe that in figure1, and figure2, A-optimality and D-optimality respectively for Self Relocating Design is minimum compared to Type II Censoring design, also in figure3 E-optimality is more for SRD. So here, we can conclude that SRD works more better than Type-II censoring on the basis of all optimality criteria.

DATA ANALYSIS ON REAL DATA

Analysis under Type II Censoring and Self-Relocating Design:

The data set- I represent the survival times of a group of patients suffering from Head and Neck cancer disease and treated using radiotherapy (RT), the data set- II represents the survival times of a group of patients suffering from Head and Neck cancer disease and treated using a combination of radiotherapy and chemotherapy (RT+CT) reported by Efron B (1988) Logistic regression, survival analysis and the Kaplan-Meier curve. Journal of the American Statistical Association.

Table6: Descriptive Statistic

Statistic	Radiotherapy	Radiotherapy + Chemotherapy
Sample Size	40	40
Range	1139	804.8
Mean	227.94	183.5
Variance	65233	37953
Std. Deviation	255.41	194.81
Coef. of Variation	1.1205	1.0617
Min	7	12.2
25% (Q1)	69	59.637
50% (Median)	151.5	115.5
75% (Q3)	266.75	205.5
Max	1146	817

From descriptive statistic patient who receives Radiotherapy treatment having average survival time 227 days and patient who receives combination of Radiotherapy and Chemotherapy treatment having average survival time 183 days. As coefficient of variation in combination of Radiotherapy and Chemotherapy treatment having lesser than that of Radiotherapy treatment indicates combine treatment has consistent effect on patients.

To check whether the data follows exponential distribution. We fit the distribution using Easyfit and from Kolmogorov-Smirnov Statistic we conclude that both datasets follow exponential distribution. The Tables for Kolmogorov-Smirnov Statistic are given below:

For data set -I,

H_0: The data follow exponential distribution.

Vs

H_1: The data do not follow the exponential distribution.

Table7: Kolmogorov-Smirnov test for data set-I

Kolmogorov-Smirnov					
Sample Size Statistic P-Value Rank	40 0.13488 0.42346 1				
α	0.2	0.1	0.05	0.02	0.01
Critical Value	0.1654	0.1891	0.2101	0.2349	0.2520
Reject?	No	No	No	No	No

Here we may conclude that the above data comes from Exponential distribution.

For data set -II,

H_0: The data follow exponential distribution.

Vs

H_1: The data do not follow the exponential distribution.

Table8: Kolmogorov-Smirnov test for data set-II

Kolmogorov-Smirnov	
Sample Size Statistic P-Value Rank	40 0.0955 0.8249 2

α	0.2	0.1	0.05	0.02	0.01
Critical Value	0.1654	0.1891	0.2101	0.2349	0.2520
Reject?	No	No	No	No	No

Here we may conclude that the above data comes from Exponential distribution.

Estimation of Parameters, Relibility and Hazard Rate under Type II Censoring and SRD

m=2, β=(0.0044, 0.0055), $\bar{F}(182)$= (0.4500, 0.3708), $\bar{F}(365)$= (0.2016, 0.1368)

Table 6: Maximum likelihood Estimate of parameters, Reliability and Hazard Rate

Design	**u**	**G**	$\hat{\beta}_1$	$\hat{\beta}_2$	$\bar{F}_1(t)$		$\bar{F}_2(t)$		**r₁(t)**	**r₂(t)**
					182	**365**	**182**	**365**		
Type II Censoring	40	20	0.0044	0.0058	0.4521	0.2035	0.3487	0.1209	0.0044	0.0058
SRD	40	40	0.0045	0.0061	0.5676	0.1961	0.4277	0.1103	0.0045	0.0061

For Radiotheorapy the probability of patient survivng for six months and one year is more as compare to combination of radiotherapy and chemotherapy. It's observed that the probability of patient surviving decreases as number of days increases.

CONCLUSIONS

Self-Relocating Design has the better performance than Type II Censoring with respect to A-optimality, D-optimality and E-optimal criterion. SRD has the better on the basis Reliability, Hazard rate and their efficiency measures as compare to Type II Censoring. From descriptive statistic on real data, patient who receives Radiotherapy treatment having average survival time 227 days and patient who receives combination of Radiotherapy and Chemotherapy treatment having average survival time 183 days. As coefficient of variation in combination of Radiotherapy and Chemotherapy treatment having lesser than that of Radiotherapy treatment indicates combine treatment has consistent effect on patients. By using Self Relocating Design and Type II Censoring the we can conclude that rate of failure of patients is less if radiotherapy (RT) is used as compare to combination of radiotherapy and chemotherapy (RT+CT) used on patient who is suffering from Head and Neck cancer disease. By using SRD and Type-II Censoring it is clear that probalility of patients surviving for six or one year is more ifradiotherapy (RT) is used instead ofcombination of radiotherapy and chemotherapy (RT+CT).

REFERENCES:

1. A.A. Dharmadhikari, A.D. Dharmadhikari, Kirti R. Shah, (2000) Comparison of two experimental designs in reliability studies, Journal of Statistical Planning and Inference, Vol 84, pp- 237–248, (ISSN - 0378-3758)
2. Amita Dharmadhikari, (2002) Planning and Analysis of Data Influenced by Two-Factors in Self-Relocating Design, American Journal of Mathematical and Management Sciences Vol 22(3-4), pp- 321-341, (ISSN - 0196-6324)
3. B. Efron, (1988), Logistic regression, survival analysis and the Kaplan-Meier curve. Journal of the American Statistical Association, 83(402) 414-425.
4. D. P. Raykundaliya, (2016), Reliability Oriented Designs, Registrar, Sardar Patel University, Vallabh Vidyanagar
5. J. N. Srivastava, (1986) Experimental designs for the assessment of reliability. In A.P. Basu (Ed.), Reliability and Quality Control, Elsevier Science Publishers, Amsterdeam,369–394.
6. J. N. Srivastava, (1987) More efficient and less time-consuming censoring designs for life testing. J. Statist. Plann. Inference, 16:389–413.

7. J. N. Srivastava, (1989), Advances in the statistical theory of comparison of lifetimes of machines under the generalized weibull distribution. Commun. Statist. - Theory Meth., 18(3):1031–1045.
8. Jayant V. Deshpande, Sudha G. Purohit, Life Time Data: Statistical Models and Methods,
9. M. R. Karim and M. A. Islam, Reliability and Survival Analysis, Springer Nature Singapore Pte. Ltd., 1-2 (2019)
10. N. B. Khoolenjani (2016), Estimating the parameter of Exponential distribution under Type-II censoring from fuzzy data, Journal of Statistical Theory and Applications, 15(2) 181-195.
11. R. Shanker, H. Fesshaye, S. Selvaraj; On Modeling of Lifetimes Data Using Exponential and Lindley Distributions. Biom Biostat Int J 2(5) 00042 (2015).
12. Shanubhogue, A., Raykundaliya, D.P., (2015a) A Study of Inferential Problem about Comparison of Lifetimes of Homogeneity of Several Systems under Generalized Exponential Distribution with Type II Censored Sampling Design, ProbStat forum, Volume-8, pp- 24–33, (ISSN 0974-3235)
13. Shanubhogue, A., Raykundaliya, D.P., (2015b) Estimation of Parameters and Reliability Characteristics in Self Relocated Design, International Journal of Statistics and Reliability Engineering Vol-2(1), pp- 18–40 (ISSN 2350–0174)
14. Shanubhogue, A., Raykundaliya, D.P., (2015c) Estimation of parameters and reliability characteristics in self-relocated design under generalized inverted scale family of distributions (GIFD), Mathematics in Engineering, Science and Aerospace, Vol-6(2), pp- 157–176, (ISSN 2041–3165)