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Review of Literature on Various Censoring Schemes under Different Lifetime Distributions

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Abstract:

In a comparative environment, both manufacturing and service industries make efforts to enhance the quality and reliability of their products through several measures, one of which is product functional lifetime. In this paper, we review the literature available on various censoring schemes under different life-time distributions. We observed various statistical estimation techniques in different censoring schemes used in the studied literature. After this study, we come to the conclusion that there are still many scopes available to study the censoring scheme for various distributions.

Keywords: Self Relocating Design, Reliability, Type-II censoring, Two-way classification, life-time distribution

1. Introduction

In previous days, the market used to be monopolistic, and consumers were forced to accept any given goods without questioning its reliability or quality. Later, as times changed, customers began to seek products with acceptable quality and reliability at a reasonable price point. The period of globalisation and liberalisation brought about a competitive environment among reliable products, retaining them at an accessible price by lowering the amount of material needed to its minimum level and offering the goods in sufficient quantities to satisfy the market's needs. The functional life-time of a product is one of its important criteria under study. A basic life testing experiment involves testing a number of things, and the recorded life time of all or some of the tested items generates the life time data. In addition to determining the warranty of the product, the data from such life-testing experiments are useful for finding reliability and failure rates. For example, the life-time of electrical appliances or automobile products may be studied in order to determine their appropriate functioning and their life-time warranties. Additionally, in a market where there is competition, ongoing product innovation becomes crucial and essential. One method for evaluating product quality and product enhancement is through life-testing experiments.

The reliability of a product whose lifetime distribution follows an exponential, a Weibull, or a generalised exponential distribution (GED) is measured using several types of censoring schemes which are discussed in the statistical literature for data collection in lifetime testing experiments. These include Type-I, Type-II, progressive, and hybrid censoring. The scale family of distributions plays a vital role in lifetime data analysis. In reliability theory, the most popular lifetime distribution is the exponential distribution due to its ease of use, and mathematical feasibility. Along with this exponential distribution, the Rayleigh distribution, the gamma distribution, and the Weibull distribution are other distributions that are widely used in reliability studies. Gupta et al. (1999) have proposed the generalized-exponential distribution (GED), which is an alternative distribution to the Weibull and gamma families of distribution for analysing lifetime data sets, and the authors studied some of its properties. The different moments have been obtained for the generalised exponential distribution using its moment-generating function. The method of maximum likelihood, method of moment, percentile method, least square method, and Lmoment method have been discussed by Gupta and Kundu (2007). Mohammad Z. Raqab (2008) discussed the estimation of the stress-strength parameter of that system, $P(Y<X)$, which was obtained by using the modified maximum likelihood estimation method and Bayesian techniques on the basis of complete samples.

In Section 2, we briefly review various censoring schemes discussed in many works of literature. In Section 3, we discussed several probability distributions of life-time data. In Section 4, we suggested different methods of estimating the parameters of the distributions under study. The concluding remarks are given in Section 5.

2. Censoring Schemes

Epstein (1954, 1960) proposed "a mixture of Type-I and Type-II censoring schemes known as a hybrid censoring scheme." The fact that standard Type-I, Type-II, or hybrid censoring techniques do not permit the removal of units at points other than the experiment's endpoint is one of its disadvantages. This limitation has been overcome in the Type-II progressive censoring design proposed by Cohen (1963). As an example, imagine that n units are used in a life-testing experiment, and the experimenter chooses in advance the quantity m as the number of failed units. Now, R_1 of the remaining n - 1 functioning units are withdrawn at random from the experiment at the moment of the first failure. Moving on, R_2 of the remaining n - R_1 - 2 units are withdrawn at random from the experiment at the moment of the second failure. Continuing in this way finally, all of the remaining $n - m - R1 - \cdots - Rm-1 = Rm$ functioning units are eliminated from the experiment at the moment of the m^{th} failure. The statistical inference on the parameters of failure time distributions under progressive type II censoring has been studied by several authors. Mann (1971), Viveros and Balakrishnan (1994), and Hofmann et al. (2005) demonstrated that type-II progressive censoring methods frequently outperform type-II conventional censoring. For more details on progressive censoring, refer to Balakrishnan and Agarwala (2000) and Balakrishnan (2007). B. Pradhan and D. Kundu (2009) focused on the estimation of parameters of the generalized-exponential distribution under progressive censoring using maximum likelihood estimators (MLEs) using the EM algorithm. Ng et al. (2009) "introduced an adjustment of the Type-II progressive hybrid censoring scheme, the so-called adaptive Type-II progressive hybrid censoring (APHC) scheme, and analyzed the data under the assumptions of an exponential lifetime distribution of the experimental units."

For more than two samples, Srivastava (1986) generalized the Type II censoring scheme. Self-relocating design (SRD) was first introduced and developed by Srivasatva (1986, 1987, 1989). The SRD is an alternative to current censoring methods that collect performance data on a variety of brands, presumably in a variety of settings. In the traditional type I and type II censorship studies, one must run m distinct experiments for each brand. If m different brands are to be tested under study in n different environmental conditions, then a total of mn number of independent experiments need to be conducted. Under SRD, all m brands are to be tested jointly under all n environmental conditions; failed units are replaced by new units of the same brand; and some more units are added (indicating by 1) or removed (indicating by 2) once a failure occurs. The added no. of units either equals (1) in each failure or is random (2). Thus, the experiments having data among themselves on the no. of units have tested for a specific combination of brand and environmental condition would depend on how failures have occurred in other experiments.

Srivastava (1986) has developed a one-way analysis of SRDllE and Srivastava (1987, 1989) has developed a one-way analysis of SRD21W. He has used brands of units as the basis of classification. Further, Srivastava and Siddiqui (1987) have studied the performance of these designs under the A-optimality and D-optimality criterias. Recently, Shanubhogue and Raykundaliya (2015b) did a comparative study of self-relocating design (SRD) and Type-II censoring design when lifetime follows the GED defined by Gupta and Kundu (1999) and the generalised inverted family of distribution proposed by Potdar and Shirke (2013) in one-way classification. In the two-way classification comparative study between SRD and Type-II censoring discussed by Dharmadhikari et al. (2000), lifetime follows exponential and Weibull distributions.

3. Probability Distributions of Life Time Data under Various Censoring Scheme

According to the lifetime research, the exponential distribution is more common. On a similar line, the three parameters in the Weibull and gamma distributions are also the most popular to study the lifetime data. Rameshwar D. Gupta and Debasis Kundu (1999) introduced the generalized-exponential distribution (GED), which is an alternative distribution to the Weibull and gamma families of distribution for analyzing lifetime data. While analyzing the censored data by using GED, the inference can be done easily as compared to the gamma family since the distribution function of GED is in a closed form. K. G. Potdar and D. T. Shirke (2013) studied a generalized inverted scale family of distributions. Some members of this family have observed such as, scale family distribution, exponential distribution, Rayleigh distribution and generalized inverted half-logistic distribution. A.A. Dharmadhikari et al. (2000) have considered two-way classification under the multiplicative model when the life time of the data follows an exponential distribution. Amita Dharmadhikari (2002) considered two different factors, the "brand of units" and the "environment," to test the main and interaction effects of the two-factor design. In this study, MLEs of different effects are obtained, their information matrix is evaluated, and LRT are proposed to test whether the brands, environments, and their interactions are significantly different. MLE and LRT statistic are evaluated by considering a randomized block design setup. On the basis of the expected overall testing duration, a further comparison of SRD and Type-II censoring has been considered, expected cost for two designs, and optimality criteria. The author suggested SRD gives a better result as compared to Type II design. That is, SRD may require less expected time and cost for testing. Anwar M. Hossain et al. (2003) conducted a comparison between the maximum likelihood (ML) and least square (LS) methods for estimating the parameters of a two-parameter Weibull distribution using complete sampling and type-II censoring. The bias and mean squared error of the estimators are compared. Through the use of simulation, the effectiveness of the ML and LS estimators is contrasted. The ML approach of estimate appears to work more well. Rameshwar D. Gupta et. al. (2003, 2004) and Debasis Kundu et. al. (2005) has discriminated the Generalized-Exponential Distributions with Weibull, Gamma, and log-normal distributions. Rameshwar D. Gupta and Debasis Kundu (2007) suggest the GED is the best alternative instead of the gamma, Weibull, and lognormal distributions. GED has a unimodal density function that is skewed, right-tailed, and has a monotone hazard function similar to the Weibull or gamma density function. Also, the author suggested that the introduction of a location parameter is possible, and not much work has been done on the three-parameter GED. So, there is scope to do a study on the three parameters of the GED.

4. Different Methods of Estimation of Parameters of the Distributions Under Study

Shanubhogue A., and Raykundaliya D. P. (2015b, 2015c) considered simultaneous estimation of parameters of GED and GIFD under the SRD or Type-II censoring design through the maximum likelihood estimation method. Since a closed form of the estimator is not feasible in this situation, the estimates are computed using the Newton-Raphson technique. Additionally, using the invariance property of MLE, reliability and failure rate functions derived by the Monte Carlo simulation technique. The effectiveness of the estimations made using the biasedness and MSE criteria. Additionally, a likelihood ratio test is used to determine the homogeneity of various system types. Moreover, in this study, performance measures like duration and total time on test, cost of experiment, and design optimality criteria were discussed. B. Pradhan and Kundu (2009) considered the EM algorithm to estimate the MLE of unknown shape and scale parameters of the GED under progressive censoring. They also computed the expected Fisher information matrix using the missing value principle. Hare Krishna and Kapil Kumar

(2013) discussed the MLE method and the least square method for obtaining the unknown parameters of a generalized inverted exponential distribution on the basis of progressively type II censored sample data. In this study, the author suggested the “least square method quite satisfactory and better than MLE. For the life-time testing experiment, Ng H. K. T. et. al. (2009) “introduced an adaptive Type-II progressive censoring scheme for exponentially distributed failure times.” Here, point estimation and six different techniques for constructing the interval (CI) of the failure rate are discussed. such as conditional exact CI, normal approximation to MLE, normal approximation of log-transformed MLE, likelihood ratio-based CI, bootstrap CI, and Bayesian analysis. Further expected total time for experimentation is explained. Comparisons between various methods of constructing CI were done using the Monte Carlo simulation technique.

5. Conclusion

In this paper, we found that, for life-time testing experiments, in many statistical literature, various censoring schemes have been studied, such as Type-I censoring, Type-II censoring, hybrid censoring, Type-II progressive censoring, generalized Type II censoring, and self-relocating design under different life-time distributions like exponential, gamma, Weibull, log-normal, and GED. The comparative study carried out in some literature, where comparisons were done on two different lifetime distributions of skewed data. Rameshwar D. Gupta and Debasis Kundu (2007) concluded that the GED is superior to the gamma, Weibull, and lognormal distributions for life-time data. GED has a unimodal density function that is skewed, right-tailed, and has a monotone hazard function has same as like Weibull, gamma, or lognormal density functions.

A comparative study of two censoring design has been studied by some researchers. A.A. Dharmadhikari et al. (2000) “comparison of SRD and Type II censoring has been explored in terms of expected total time for the test, expected costs for the two designs under consideration, and optimality criteria of the design.” According to this study, SRD can be less expensive and less time-consuming than type II censoring scheme. Shanubhogue and D. P. Raykundaliya (2015c) investigated SRD and generalized type II censoring scheme for the lifetime data of the generalized inverted scale family of distributions (GIFD). Shanubhogue A. and Raykundaliya D. P. (2015b) investigated SRD and generalized type II censoring scheme for a generalized exponential distribution of lifetime data. According to the author, in terms of convergence to their real values, the MLE technique works well. Whenever the shape parameter is not known and the GIFD of lifetime is taken into account, the SRD design is A-optimal, D-optimal, and E-optimal in comparison to a generalized Type II censoring design for GIFD. But the SRD design is always E-optimal and is A-optimal and D-optimal when the shape parameter is unknown, while lifetime is GED considered as compared to a generalized Type II censoring design. Further, the SRD is reducing the experiment's expense in comparison to generalized Type II censoring in both studies.

On the line of Shanubhogue and Raykundaliya (2015a, 2015b, and 2015c) and Dharmadhikari et. al. (2000) we can study comparison of SRD and Type-II censoring when lifetime follows GED in two-way classification by considering one factor as brand of the product and other is environmental effect.

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